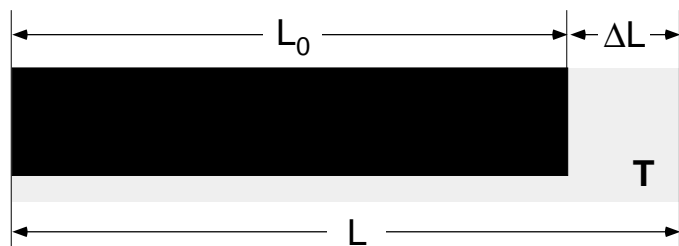


# PhyzGuide: Thermal Expansion

how and why "they plump when ya cook 'em"

## THE HOW: THERMAL EXPANSION IN THE MACROSCOPIC VIEW

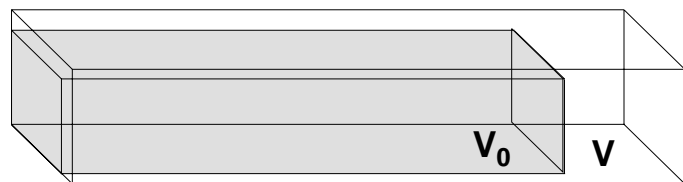
**1. Linear expansion in solids.** A bar of length  $L_0$  at temperature  $T_0$  expands to a length  $L$  when heated to a temperature  $T$ . The change in length  $\Delta L$  (i.e.  $L - L_0$ ) is related to the change in temperature  $\Delta T$  (i.e.  $T - T_0$ ) and the original length  $L_0$  by a constant that depends on the material. This constant is the coefficient of linear expansion (also called the coefficient of thermal expansion),  $\alpha$ .



The black rectangle indicates the size of the bar at the low temperature  $T_0$ . The gray region shows the expansion that occurs upon warming to  $T$ . This growth is greatly exaggerated in the diagram. If the bar were cooled back down to  $T_0$ , its size would return to that indicated in black.

- The change in length is directly proportional to the change in temperature:  $\Delta L \propto \Delta T$ .
- The change in length is directly proportional to the original length:  $\Delta L \propto L_0$ .
- The change in length is directly proportional to the coefficient of linear expansion:  $\Delta L \propto \alpha$ .
- Together, these proportionalities form the equation  $\Delta L = \alpha L_0 \Delta T$ .

**2. Volume expansion in solids and liquids.** A volume  $V_0$  of solid or liquid at temperature  $T_0$  expands to a volume  $V$  when heated to a temperature  $T$ . The change in volume  $\Delta V$  (i.e.  $V - V_0$ ) is related to the change in temperature  $\Delta T$  (i.e.  $T - T_0$ ) and the original volume  $V_0$  by a constant that depends on the material. This constant is the coefficient of volume expansion,  $\beta$ .



The gray parallelepiped indicates the size of the solid or liquid at the low temperature  $T_0$ . The transparent region shows the expansion that occurs upon warming to  $T$ . This growth is greatly exaggerated in the diagram. If the volume were cooled back down to  $T_0$ , its size would return to that indicated in gray.

- The change in volume is directly proportional to the change in temperature:  $\Delta V \propto \Delta T$ .
- The change in volume is directly proportional to the original volume:  $\Delta V \propto V_0$ .
- The change in volume is directly proportional to the coefficient of volume expansion:  $\Delta V \propto \beta$ .
- Together, these proportionalities form the equation  $\Delta V = \beta V_0 \Delta T$ .
- For solids whose coefficient of linear expansion is known, the equation  $\Delta V = 3\alpha V_0 \Delta T$  may be used.

**3. Volume expansion in gases.** The volume  $V$  of a gas is related to its temperature and the pressure at which it is contained. Raising the temperature of the gas from  $T_0$  to  $T$  increases the volume from  $V_0$  to  $V$  if the pressure  $P$  is held constant (Charles' Law). Reducing the pressure from  $P_0$  to  $P$  of a gas increases the volume of a gas from  $V_0$  to  $V$  if the temperature is held constant (Boyle's Law). Taken together, these proportionalities form Charboyle's Law (a.k.a. The Ideal Gas Law):  $P_0 V_0 / T_0 = PV / T$ .

You dealt with this in chemistry. The "Why" of this equation is The Kinetic Theory of Gases.

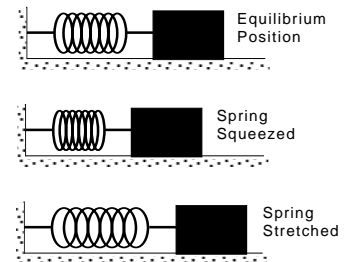
OK, the "Charboyle" bit is an embellishment.

## THE WHY: THERMAL EXPANSION IN THE MICROSCOPIC VIEW

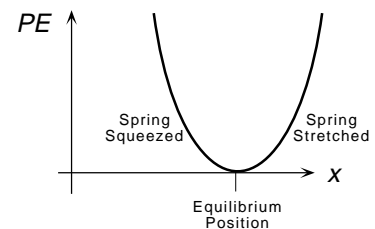
Why do solids expand the way they do under an increase in temperature? The common explanation is that because the particles (atoms and molecules) jiggle more rapidly, atoms are pushed away from each other. This is a simplification that may suffice as a “first pass” explanation, but it breaks down shortly thereafter. I will offer a more complete explanation of why thermal expansion occurs, but I do not expect you to master it. You should be able to read it and answer a few simple questions about it.

**The Potential Well.** Recall that potential energy is energy of position, and that potential energy is at a minimum when an object is in stable equilibrium.

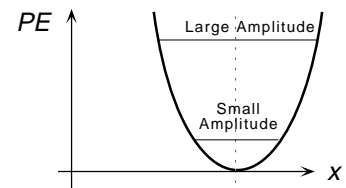
Consider a mass attached to a spring that is, in turn, attached to a wall. The mass is in stable equilibrium. When the spring is both unstretched and uncompressed, the spring exerts no force on the mass—the mass has the minimum potential energy possible. If the mass is moved toward the wall, the spring is compressed and exerts an outward force on the mass—the potential energy of the mass is increased. If the mass is pulled away from the wall, the spring is stretched and exerts an inward force on the mass—the potential energy is again increased above the minimum.



A graph could be constructed to show the potential energy of the mass *vs.* distance from the wall. Because potential energy is always measured with respect to an arbitrary (not absolute) reference point, the potential energy at equilibrium could be chosen as positive, negative, or zero. Whatever it is chosen to be, it is at a minimum value when the mass is at the equilibrium position. The *amplitude* of an oscillation can be taken as the horizontal distance between two points of equal potential energy on the graph.

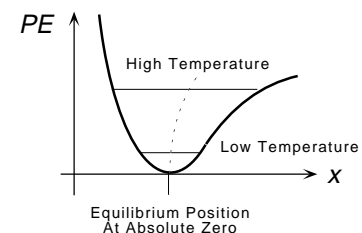


Since the shape of the graph is like that of a hole dug in the ground, such a plot is sometimes called a *potential well*. For the ideal spring-mass system we constructed, the potential well is symmetric. No matter how large the oscillations become, the equilibrium position is always the same distance from the wall. The “average” location of the mass as it slides back and forth is the equilibrium position. So small oscillations or large, the mass is always—on average—the same distance from the wall.



**Asymmetry in the Potential Well.** Adjacent atoms or molecules in a solid typically behave as if they were connected by tiny springs. There is an equilibrium distance at which the particles have minimal potential energy. If the particles are pushed closer together, they repel and potential energy is increased. If the particles are pulled farther apart, they attract, and again potential energy is increased. Due to the nature of the interatomic or intermolecular forces, however, the graph is not exactly symmetric. Thus there is *asymmetry* in the potential well.

As the kinetic energy of particles in a solid increases, they jiggle with greater and greater amplitude. The amplitude of an oscillation is still taken to be the horizontal distance between two points of equal potential energy on the graph. But notice what happens to the average location (indicated by the dotted path on the graph) as the amplitude of the oscillations increases. Due to the asymmetric nature of the potential energy curve, it occurs at an increasing distance of particle separation.



So as the particles in the substance move with more and more kinetic energy, the spacing between the particles increases. The atoms don't get any larger. The molecules don't necessarily get any larger. But the average distance between the atoms or molecules does get larger. The relation between temperature and interparticle distance is nearly linear, and can therefore be approximated with great accuracy by the equations given in parts 1 and 2 on the other side.