PhyzExamples: Heat Equations

Physical Quantities • Symbols • Units • Brief Definitions

Internal Energy • *U* • joule: J • Energy associated with the random motions and relative locations of the particles (atoms/ molecules) of a body.

Heat • *Q* • joule: J • Internal energy that is transferred between bodies, typically from hotter bodies to colder bodies.

Thermal Conductivity • *k* • W/m·°C • A measure of how readily heat can pass through a material.

Emissivity • *e* • unitless • A measure of a surface's ability to radiate ranging from 0 (non-radiator) to 1 (perfect—or *blackbody*—radiator).

Stefan-Boltzmann Constant • $\sigma = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}$.

Heat Capacity • *C* • J/°C • The amount of heat added to a body to raise its temperature by a given quantity. Typically, the heat that will raise the temperature of a body by one Celsius degree. Also the amount of heat removed to cool the body by one degree.

Specific Heat Capacity • *c* • J/kg·°C • The amount of heat needed to raise the temperature of a specific mass of a substance by a given quantity. Typically, the heat added to one kilogram of a substance to raise its temperature by one Celsius degree. Also the heat removed from one kilogram of a substance to reduce its temperature by one kilogram by one Celsius degree.

Heat of Fusion • L_f • J/kg • The amount of heat needed to transform a specific amount of a substance from solid to liquid. Typically, the amount of heat added to one kilogram of solid to melt it. Also the heat removed from one kilogram of liquid to freeze it.

Heat of Vaporization • L_v • J/kg • The amount of heat needed to transform a specific amount of a substance from liquid to gas. Typically, the amount of heat added to one kilogram of liquid to vaporize it. Also the heat removed from one kilogram of gas to condense it.

Equations

$Q/t = kA\Delta T / d$ • *heat conduction rate = thermal conductivity · cross sectional area · temperature difference (between hot and cold regions) / thickness of conducting material*

$Q/t = \sigma e A T^4$ • *radiation rate = Stefan-Boltzmann constant · emissivity · surface area · absolute temperature to the fourth power*

$C = mc$ • *heat capacity = mass · specific heat capacity*

$Q = mc\Delta T$ • *heat added = mass · specific heat capacity · change in temperature*

$Q = mL_f$ • *heat added = mass · heat of fusion*

$Q = mL_v$ • *heat added = mass · heat of vaporization*

PhyzExamples: Heat Examples

Smooth Operations Examples

1. How much heat is lost through conduction in one hour from a log cabin whose walls and ceiling are 30cm thick? The conducting area of the cabin is 250m^2. The inside temperature is 20°C and the outside temperature is –10°C.

1. $t=3600s$ (one hour) $d=0.3m$ $A=250m^2$ $\Delta T=30°C$ (20– –10) $k=0.13W/m{\cdot}°C$

$Q/t = kA\Delta T/d$

$Q = kA\Delta Tt/d = 0.13W/m{\cdot}°C \cdot 250m^2 \cdot 30°C \cdot 3600s / 0.3m$

<u>$Q = 11.7x10^6J = 11.7MJ$</u>

2. The University of Michigan's Regents' Plaza is home to a giant rotating metal cube*. The cube is painted black and measures 2m along each edge. It is supported on—and rotates about—one of its eight corners. At what rate does it radiate thermal energy at night if its temperature is 17°C?

2. $A=6x(2m)^2=24m^2$ (cube) $e=1$ (black) $T=17°C+273°C=290K$

$Q/t = \sigma eAT^4 = 5.67x10^{-8}W/m^2{\cdot}K^4 \cdot 1 \cdot 24m^2 \cdot (290K)^4$

<u>$Q/t = 9620W$</u>

**Sometimes the college kids like to dress it up with big white dots so it looks like a die. Or they'll adhere colored squares to it so it looks like a giant Rubic's Cube®. Those wacky wolverines, I tell ya.*

3. What is the specific heat of a substance if 37kJ of heat gives rise to an 8.2°C rise in the temperature of a 23kg sample?

3. $Q=37{,}000J$ $\Delta T=8.2°C$ $m=23kg$ $c=?$

$Q = mc\Delta T$

$c = Q/m\Delta T = 37{,}000J / (23kg)(8.2°C)$

<u>$c = 196J/kg°C$</u>

4. How much energy is given off when a 45kg iron ingot at 800°C is quenched in water to a final temperature of 80°C?

4. $m=45kg$ $\Delta T=-720°C$ $c=460J/kg°C$

$Q = mc\Delta T = (45kg)(460J/kg°C)(-720°C)$

<u>$Q = -14.9x10^6J = 14.9MJ$ given off</u>

5. How much heat is needed to melt a 50g sample of silver at its melting point?

5. $m=0.05kg$ $L_f=88{,}000J/kg$

$Q = mL_f = 0.05kg \cdot 88{,}000J/kg$

<u>$Q = 4400J$</u>

6. How much water at 100°C could be vaporized by the addition of 1MJ of heat?

6. $Q=1x10^6J$ $L_v=2.26x10^6J/kg$

$Q = mL_v$

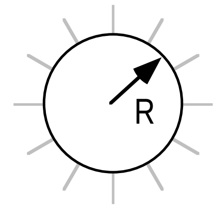
$m = Q/L_v = 1x10^6J / 2.26x10^6J/kg$

<u>$m = 0.44kg$</u>

Welcome to the Real World Examples

7. A star with a radius of 2.43×10^9m is found to be radiating energy at the rate of 3.41×10^{26}W. Assume it is a blackbody radiator and determine its surface temperature.

7. $R = 2.43 \times 10^9$m $Q/t = 3.41 \times 10^{26}$W   e=1 (blackbody)   T=?



$Q/t = \sigma e A T^4$

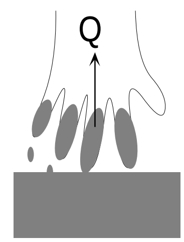
$A = 4\pi R^2$ (surface of a sphere)

$T = \sqrt[4]{[(Q/t) / (\sigma e 4\pi R^2)]}$

$T = \sqrt[4]{[(3.41 \times 10^{26}\text{W}) / (5.67 \times 10^{-8}\text{W/m}^2{\cdot}\text{K}^4 \cdot 1 \cdot 4\pi(2.43 \times 10^9\text{m})^2]}$

$\underline{T = 3000\text{K}}$

8. Jearl sometimes likes to dip his fingers into molten lead. Among the many dangers of this stunt is that of the lead (at 330°C) “freezing” onto his fingers. If 400g of lead at its melting point were to freeze on Jearl’s hand when he dipped it in,
a. how much heat would be transferred to Jearl’s innocent flesh?
b. what rise in temperature would occur in Jearl’s tender skin assuming the lead affected 100g of flesh and the specific heat of the affected outer flesh is 1kJ/kg·°C? (Note this is a lower value than for the human body since the human body is primarily water while the outer layers of skin contain less water per given volume.)



8. $m_L = 0.4$kg $L_f = 25{,}000$J/kg $m_F = 0.1$kg   c=1000J/kg·°C

a. $Q = mL_f$

$Q = 0.4\text{kg} \cdot 25{,}000\text{J/kg}$

$\underline{Q = 10{,}000\text{J}}$

b. $Q = m_F c \Delta T$

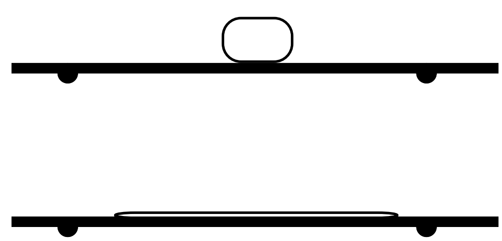
$\Delta T = Q / m_F c$

$\Delta T = 10{,}000\text{J} / (0.1\text{kg} \cdot 1000\text{J/kg}{\cdot}°\text{C})$

$\underline{\Delta T = 100°\text{C}}$ <u>(that would burn him!)</u>

9. Suppose a Miracle Thaw® plate can melt an ice cube in 2 minutes and does so only by conduction. The ice is at 0°C and the air below the plate is at 22°C. The ice cube is 2cm on each edge and the plate is 3mm thick. What is the thermal conductivity of the plate? (Ice has a density of 917kg/m³.)

9. t=120s $\Delta T = 22$°C   x=2cm   d=0.003m $D_{ICE} = 917\text{kg/m}^3$



Heat to melt ice cube:

$Q = mL_f$

$m = DV = Dx^3$

$Q = Dx^3 L_f$

Heat conducted:

$Q = kA\Delta T t/d$

$A = x^2$

$kx^2 \Delta T t/d = Dx^3 L_f$

$k = D x L_f d / \Delta T t$

$k = (917\text{kg/m}^3)(0.02\text{m})(335{,}000\text{J/kg})(0.003\text{m}) / (22°\text{C})(120\text{s})$

$\underline{k = 6.98\text{W/m}{\cdot}°\text{C}}$ <u>(Is this number truly “miraculous”?)</u>