Garrido and Hurtado Reply: In this Reply, we answer the Comment by Li et al. [1] on our Letter “Simple One-dimensional Model of Heat Conduction which Obey Fourier’s Law” [2]. In that Letter, we studied the conductivity of a one-dimensional chain of $N$ hard-point particles with alternating masses, proving that Fourier’s law holds in this system.

First, Li et al. study the conductivity $\kappa(N)$ as a function of the system size $N$. They conclude that, as they do not observe saturation of the curve $\kappa(N)$ even for $N = 8000$ (instead, they fit $\kappa(N) \sim N^{0.33}$ for large $N$), the large-$N$ limit of $\kappa(N)$, $\kappa_\infty$, must be divergent. In contrast, our conclusion was that finite size effects on $\kappa(N)$ were too strong to conclude on $\kappa_\infty$. In particular, we were able to fit both divergent and saturating laws to our data, thus demonstrating that this kind of study does not yield conclusive information on the value of $\kappa_\infty$. Other authors have arrived at the same conclusions [3]. Moreover, we can argue [2] that $\kappa(N) = \kappa_\infty - AN^{-0.3}$, so $N = 8000$ is still far away the asymptotic region. Hence, the fact that $\kappa(N)$ in Li et al. data has not yet saturated for $N = 8000$ does not involve a divergent conductivity.

In their Comment, Li et al. also report measurements on the total energy current self-correlation function, $C(t)$. They observe that $C(t) \sim t^{-\delta}$, with $\delta \leq 0.67$, thus concluding, via the Green-Kubo formula, that $\kappa_\infty$ is divergent. However, $C(t)$ also shows important finite size effects [4]. In particular, two different long time regions appear, the first one decaying algebraically as $C(t) \sim t^{-1-\Delta}$, with $\Delta = 0.3$, and the latest one decaying as $C(t) \sim t^{-\delta}$, with $\delta = 0.88$. It can be shown [4] that the first region (which yields a finite conductivity) is the relevant one in the thermodynamic limit, being the very long time decay $C(t) \sim t^{-\delta}$, a result of finite size effects. It can also be shown that, contrary to the Li et al. claim, a close relation between $C(t)$ and the local energy current self-correlation, $c(t)$, exists; and it implies a common long time behavior of $C(t)$ and $c(t)$ for our alternating masses system [4]. This system is believed to be ergodic [3]. Thus, it is hard to believe that a change on the initial condition can yield a different decay exponent for $C(t)$ measured in equilibrium, as Li et al. claim.

Let us now speak about the energy partition between light and heavy particles. Contrary to Li et al. claims, our results on this matter do not disagree with those of Kato and Jou [5]. They found that, at the nonequilibrium stationary state of the open system, the average energy stored in heavy particles exceeds the average energy stored in light ones. On the other hand, in our Letter we studied how an energy pulse propagates through an isolated system, where there was no boundary thermalization. We observed how light particles respond dynamically to such perturbation, storing more energy on average than heavy particles.

We think that both pictures are compatible (as Li et al. confirm in their figure), and reflect the nontrivial behavior of this system. The important fact is that our system responds to the perturbation trapping energy temporarily in light particles, thus allowing a diffusive transport of energy through the system. Furthermore, we have measured the mean square displacement of the energy distribution at each time, $s(t)$. In Fig. 1, we plot $s(t)$ as a function of time, and we observe that, after an initial ballistic regime, the energy propagates diffusively, $s(t) \sim t$, and thus Fourier’s law holds in this system.

In conclusion, we think that Li et al. have analyzed their data in an erroneous way. Hence, we firmly support our previous results [2], i.e., that our one-dimensional system, which is momentum conserving and has a nonzero pressure, has a finite thermal conductivity in the thermodynamic limit, thus obeying Fourier’s Law.

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