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Driven lattice gases: new perspectives

F. de los Santos [1], P.L. Garrido

Intitute Carlos I of Theoretical and Computational Physics, Facultad de Ciencias, Universidad de Granada, E-18071 Granada, Spain

Abstract

We report analytical studies of a series of driven systems: the driven lattice gas model, the randomly driven lattice gas, the two-temperature model and the driven bi-layer lattice gas. All of them are described within a unified framework that preserves the dynamical specifications present at the discrete level. Thus, we provide a set of Langevin equations for these driven systems that illustrate how some microscopic details can affect the macroscopic properties. © 1999 Elsevier Science B.V. All rights reserved.

1. Introduction

The physics of Lattice Gases driven out of equilibrium by an external field is a subject of an ongoing interest. On one hand, it constitutes a large window into the field of non-equilibrium steady states. On the other, the Driven Lattice Gas model (DLG) and many of its relatives exhibit rich behaviour although its microscopic specifications are utterly simple. First devised by Katz et al. [1], the DLG consists of a d-dimensional (hyper)cubic lattice with periodic boundary conditions, coupled to a thermal bath at temperature T. Each lattice site is labeled by an occupation variable which takes the value 1 or 0 depending on whether it is occupied by a particle or empty, respectively. Neighboring particles attract each other according to an Ising Hamiltonian. The model is also endowed with a particle hopping dynamics which depends on differences in energy, ΔH, and an external uniform driving force $\mathbf{E}$ through $[(\Delta H + \ell E)/T]$, where $\ell = 1$ (-1) for jumps along (against) $\mathbf{E}$ and 0 otherwise. Most Monte Carlo simulations deal with systems at half filling and infinite field, which in practice means that no jumps against the field are allowed. At temperature $T^* \simeq 1.4 T_c$, T_c being the Onsager critical temperature, the DLG undergoes a non-equilibrium phase transition from a disorded state to one with spatial structure characterized by a striped particle-rich region parallel to the drive. We shall not dwell any further on the system properties and refer the reader to [2,3] for recent reviews.

2. The Langevin approach

Commonly, it is only after continuum approaches that analytical understanding in lattice systems arises, because continuous descriptions can be expected to be amenable to simpler mathematical treatment. Unfortunately, pitfalls plague the rigorous continuum route making it an insurmountable task in practice. Instead, phenomenological approaches are in order and equations of Langevin can be postulated based on global symmetries and conservation laws. This is the usual avenue followed when studying the DLG in the long-time and large-scale limit. However, this procedure results in a Langevin equation which is identical for all choices of the microscopic rules. A second objec-

[1] E-mail: disantos@onsager.ugr.es.

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tion can be raised to the standard continuum approach. Time evolution in driven systems involves the driving force and differences in internal energy. These key ingredients are mapped to a current term plus a Model B [4], respectively, at the level of the Langevin equation. Nevertheless, mindful of the subtleties present in non-equilibrium systems we cautiously give up this sort of linearity assumption and introduce these two ingredients from the very beginning, i.e., in a continuous master equation. Then, their final form in the Langevin equation is left to the calculus rather than to preconceived notions. When followed, this route leads to a Langevin equation for the DLG that reads (see [5,6] for a detailed explanation of the intermediate stages)

$$\partial_t \phi(\mathbf{r},t) = \sum_a \nabla_{\mathbf{r}_a} \big[h(\Lambda_a) + e(\Lambda_a)^{1/2} \zeta_a(\mathbf{r},t) \big]. \quad (1)$$

Here $\phi(\mathbf{r},t)$ is the coarse grained excess particle density field, a stands for the **a** direction,

$$\Lambda_a = \mathbf{a} \cdot \mathbf{E}(1-\phi^2) - \nabla_{\mathbf{r}_a} \frac{\delta \mathcal{H}}{\delta \phi}$$

and ζ_a is a Gaussian white noise. $\mathcal{H}$ is the familiar ϕ-four Hamiltonian and the functions $h(x)$ and $e(x)$ are defined as

$$h(x) = \int_R d\eta\, f(\eta)\eta w(\eta x),$$

$$e(x) = \int_R d\eta\, f(\eta)\eta^2 w(\eta x), \quad (2)$$

$f(\eta)$ being an even function of η and w the transition rates per unit time (for instance $w(x) \propto 1 - \tanh(x/2)$).

Contrary to previous proposals, the dependence of (1) on microscopic details is apparent. This becomes a weighty consideration as one moves to non-equilibrium scenarios. Also remarkably, Eq. (1) contains the basic symmetries of the DLG: it is invariant under translations in space and time, and it is also invariant under the simultaneous change $E \to -E$ and $\phi \to -\phi$. So, it constitutes a continuous counterpart to the DLG in the same sense as the standard approach [7] does. Now, after discarding irrelevant terms in the renormalization group sense by naive power counting in the Langevin equation (1) our major first result emerges, namely, that the E-infinite case belongs to a different universality class than the E-finite case [5,6].

Table 1
The three categories into which phase transitions in the DLG, RDLG and 2T model can be classified

A	B	C
2T ($T_\parallel = \infty$)	2T ($0 < T_\parallel < \infty$)	DLG ($0 < E < \infty$)
RDLG ($E = \infty$)	RDLG ($0 < E < \infty$)	
DLG ($E = \infty$)		

For the former, the upper critical dimension follows as $d_c = 4$ while a dimensional shift to $d_c = 8$ occurs in the latter. The crossover phenomenon between these two different critical behaviors can be understood on the basis of an anisotropic structure factor. A renormalization group analysis of the infinitely driven system is possible and it yields an universality class other than that obtained from the equation postulated in [7]. The value of the critical exponents found after a calculation to one-loop order in $\varepsilon = 4 - d$ seem to be consistent with the simulation ones [8]. The occurrence of microscopic dynamics in (1) is instrumental in getting these novel results. Numerical work is in progress to investigate the failure of the Cahn–Hilliard approach to coarsening dynamics [9] from this new perspective.

It is important to find out whether other driven systems can be incorporated within the formalism described above. To this end, we have studied the *randomly driven lattice gas* (RDLG) [10], the *two-temperature model* (2T) [11] and the *driven bi-layer lattice gas model* [12], and coherently classified the existing phase transitions in these systems [6]. More precisely, the infinitely driven DLG, the RDLG with infinite drive and the two-temperature model with $T_\parallel = \infty$ share the same Langevin equation. The RDLG with finite average field and the two-temperature model with finite $T_\parallel$ belong to the same universality class. The DLG with $E < \infty$ is a class by itself (see Table 1). Finally, the two phase transitions present in the driven bi-layer lattice gas can also be placed into a coherent analytical context.

3. Conclusions

We have reported a number of analytical results concerning driven, conservative, lattice systems. A carefully derived Langevin equation for the DLG has been presented. As a main consequence, a clear-cut distinction between the $E = \infty$ and the E finite cases emerges: they belong to different universality classes. Work is in progress to clarify some issues concerning asymmetric growth during phase segregation. Our formalism also encompasses several extensions of the DLG: the RDLG, the two-temperature lattice gas and the driven bi-layer lattice gas. It follows that a coherent classification of the phase transitions in theses systems can be provided in the frame of field theory. Many interesting questions await more detailed theoretical investigations, but our preliminary results already indicate how in some non-equilibrium systems the details of the microscopic dynamics can be decisive to the observable behavior.

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