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Nonequilibrium lattice models: A case with effective Hamiltonian in *d* dimensions

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The steady-state configurational distribution of an Ising-type family of competing dynamics lattice models is explicitly found for any dimension. These models are characterized by a spin-flip dynamics which is a linear superposition of transition rates. Each individual rate attempts to drive the system asymptotically to a different equilibrium state. In general, the stationary distribution of this kind of model is not known for dimensions higher than 1. However, for a particular type of rate, we show that the stationary state is a Gibbsian one with an effective Hamiltonian whose coupling constants depend on the details of the dynamics. As an application, some related magnetic impure models are studied.

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Lattice models are a natural way for understanding complex real systems. They are intended to simulate the behavior of many systems in physics, biology, chemistry, sociology, etc. [1]. Usually lattice models are defined following two different strategies: (a) by giving an explicit form of the interaction Hamiltonian, and (b) by giving a set of local dynamic rules. In the first case, the stationary-state distribution is known to be a Gibbsian one, and it can be analyzed in the context of the equilibrium statistical mechanics. The second case is more general in the sense that the stationary distribution, if it exists, is not known *a priori* and does not have to be a Gibbsian one. Commonly, real systems are in nonequilibrium states, that is, some external agent prevents them from reaching an equilibrium distribution. Therefore we are confined to the second strategy if we want to study this kind of complex situation. In this paper we show that for a particular case of an Ising-like family of nonequilibrium lattice models we are able to find analytically its stationary distribution which is characterized by an effective Hamiltonian for any dimension.

The lattice model is defined as follows. In each node of a d-dimensional lattice, $\mathbf{x}\in\mathbf{Z}^d$, there exists a spin variable that may get two values, $s(\mathbf{x})=\pm1$. It is assumed that the time evolution of spin variables has a Markovian stochastic nature. Therefore, the probability of finding the system at a spin configuration, $\mathbf{s}\equiv\{s(\mathbf{x})|\mathbf{x}\in\mathbf{Z}^d\}$, at time t, say $\mu_t(\mathbf{s})$, is the solution of the master equation

$$\partial_t\mu_t(\mathbf{s})=\sum_{x}[c(\mathbf{s}^x;\mathbf{x})\mu_t(\mathbf{s}^x)-c(\mathbf{s};\mathbf{x})\mu_t(\mathbf{s})] , \tag{1}$$

where $c(\mathbf{s};\mathbf{x})$ is the transition probability per unit time that the spin at site $\mathbf{x}$ flips from $s(\mathbf{x})$ to $-s(\mathbf{x})$ and $\mathbf{s}^x$ denotes the spin configuration $\mathbf{s}$ after this flip.

The model is fully defined by giving the rate $c(\mathbf{s};\mathbf{x})$. In particular, if we want to guarantee that the system stationary state, the solution of Eq. (1), is an equilibrium one, characterized by the interaction Hamiltonian

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$$H(\mathbf{s};\mathbf{J})=-\sum_{A\subset\mathbf{Z}^d} J_A s_A ,\quad s_A\equiv\prod_{x\in A} s_x , \tag{2}$$

it is sufficient to consider the rate

$$c(\mathbf{s};\mathbf{x})=\phi(\beta\Delta H(\mathbf{s};\mathbf{J})) , \tag{3}$$

where $\Delta H(\mathbf{s};\mathbf{J})\equiv H(\mathbf{s}^x;\mathbf{J})-H(\mathbf{s};\mathbf{J})$, $\mathbf{J}=\{J_A\,|\,A\subset\mathbf{Z}^d\}$, ϕ is any positive function with the detailed balance property $\phi(\lambda)=\exp(-\lambda)\phi(-\lambda)$, and β^{-1} is the temperature. In this context, a nonequilibrium system is easily built if we superpose several of these "equilibrium" rates, each of them characterized by a different $\mathbf{J}$ parameter set of values, i.e.,

$$c(\mathbf{s};\mathbf{x})=\int\left[\prod_{A\subset\mathbf{Z}^d} dJ_A P_A(J_A)\right]\phi(\beta\Delta H(\mathbf{s};\mathbf{J}))$$
$$\equiv\langle\langle\phi(\beta\Delta H(\mathbf{s};\mathbf{J}))\rangle\rangle , \tag{4}$$

where $P_A(J_A)$ is the probability distribution for the J_A coupling constant and is given by the model definition. The competition between the different dynamics makes that a kind of "dynamical frustration" appears and the stationary state is expected to be, *a priori*, a nonequilibrium one. However, in recent years some studies in these systems have revealed that for some particular one-dimensional cases it is possible to find analytically the stationary-state distribution, which is then described by an effective Ising-like Hamiltonian [2]. That is, these particular cases behave as systems at equilibrium with effective parameters, but let us remark that this is not the rule but the exception. In the attempt to get some analytic description of the system behavior at dimensions higher than 1, we have found that also for the particular function, $\phi(\lambda)=\exp(-\lambda/2)$, the stationary distribution is a Gibbsian one for any dimension and distribution of J's. It is easy to demonstrate the latter assertion by realizing that the transition rate defined in Eq. (4) can be written as

$$c(\mathbf{s};\mathbf{x})=\left[\prod_{A|x\in A} N_A\right]\exp[-\beta\Delta H(\mathbf{s};\mathbf{J}')/2] , \tag{5}$$

with

$$N_A^2=\frac{\langle\langle\sinh(\beta J_A)\rangle\rangle\langle\langle\cosh(\beta J_A)\rangle\rangle}{\sinh(\beta J'_A)\cosh(\beta J'_A)} , \tag{6}$$

$$J'_A=\frac{1}{2\beta}\ln\left[\frac{\langle\langle e^{\beta J_A}\rangle\rangle}{\langle\langle e^{-\beta J_A}\rangle\rangle}\right] . \tag{7}$$

The transition rate given by Eq. (5) verifies the detailed balance condition as the equilibrium rate in Eq. (3). Therefore the stationary distribution is $\mu_{\rm st}(\mathbf{s})\propto\exp[-\beta H(\mathbf{s};\mathbf{J}')]$. Let us mention that not only is the stationary state characterized as an effective equilibrium one but also any of its dynamic properties: relaxation towards the stationary state, decay of metastable states, etc. Finally, we would like to emphasize that this is a peculiarity of the function ϕ we have considered. For any other election we have been unable to map the system into an equilibrium one.

As a particular application of the latter result we choose the d-dimensional Ising Hamiltonian as the one in Eq. (2),

$$H(\mathbf{s};\mathbf{J})=-\sum_{|x-y|=1} J_{xy}s(\mathbf{x})s(\mathbf{y}) , \tag{8}$$

where now the set A in Eq. (4) denotes all possible different pairs of nearest-neighbor sites in the lattice. In this case the system behaves as an Ising model with a coupling constant which depends on the temperature and on the bond distribution $P_{xy}(J_{xy})$. Several interesting cases appearing in the literature can easily be worked out explicitly:

(i) *Nonequilibrium impure Ising model:* $P_{xy}(J)=p\delta(J-J_0)+(1-p)\delta(J)$, $0\le p\le 1$. This model, introduced in Ref. [3], simulates the presence of nonmagnetic impurities which distribution changes with time. In this case the effective coupling constant given by Eq. (7) is

$$J'(\beta)=(2\beta)^{-1}\ln\left[\frac{p\exp(\beta J_0)+1-p}{p\exp(-\beta J_0)+1-p}\right] .$$

In particular, $J'(\beta)=J_0/2$ when $p=\frac{1}{2}$. We see that $J'(\beta)\to J_0/2$ and pJ_0 when $\beta^{-1}\to 0$ and ∞, respectively. Therefore, for $d=1$ there is no phase transition, the critical point is at zero temperature, and all the thermal critical exponents are divided by 2 with respect to the equilibrium Ising ones. For $d\ge 2$ there is a phase transition for any p with Ising-type critical exponents, and the system magnetization saturates to 1 or -1 at zero temperature for any value of $p>0$. In particular, when $p=\frac{3}{4}$ the critical temperatures are $\beta_c J_0=0.5918\ldots$ and $0.2960\ldots$ for $d=2$ and 3, respectively.

(ii) *Nonequilibrium spin glass Ising model:* $P_{xy}(J)=p\delta(J-J_0)+(1-p)\delta(J+J_0)$, $0\le p\le 1$. This model was studied in Ref. [4] in order to analyze the effect of the antiferromagnetic bond diffusion in a spin-glass Ising model. The effective coupling constant is now

$$J'(\beta)=(2\beta)^{-1}\ln\left[\frac{p\exp(2\beta J_0)+1-p}{(1-p)\exp(2\beta J_0)+p}\right] .$$

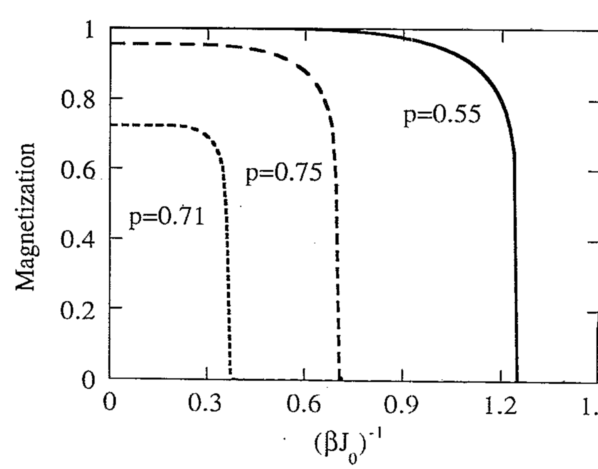

FIG. 1. Magnetization vs temperature for the two-dimensional nonequilibrium impure (solid line) and spin-glass models (dashed lines) for different p values.

In contrast to case (i), for $d=1$ and $0<p<1$ there is no critical point at zero temperature. For $d \geq 2$ there is an Ising-type critical point whenever $p>p_c(d) = [1+\exp(-2\beta_c^{(d)} J_0)]^{-1}$ or $p<1-p_c(d)$ where $(\beta_c^{(d)})^{-1}$ is the critical temperature for the Ising model at dimension d; in particular, $p_c(2)=2^{-1/2}=0.7071\ldots$ and $p_c(3)=0.6090\ldots$. In this case the magnetization does not saturate at zero temperature. When $1-p_c(d) <p<p_c(d)$ there is no phase transition. In Fig. 1 the magnetization behavior is shown for several p values and dimension 2.

(iii) *Temperature depending bond distribution:* $P_{xy}(J)=N\exp[-\alpha(J-\beta/2\alpha)^2]$, $\alpha=\alpha(\beta)$. As in the annealed models, it can be assumed that the bonds interact with the thermal bath and therefore the bond distribution may depend on the temperature. As an example, we have considered the above-mentioned temperature-dependent Gaussian distribution with $\alpha=(2\beta_c^{(d)})^{-1}\beta^2(1+c|\beta^{-1}-1|^{\gamma})$, c and γ being positive constants. In particular, if $1<\gamma<2$, the distribution average value and standard deviation go to infinity (zero) for $\beta \to 0$ (∞). That is, the characteristic bond strength and the fluctuations around it grow with the temperature. For this choice the critical temperature is 1 and the thermal critical exponents are the Ising ones multiplied by γ in all dimensions.

In conclusion, contrarily to what happens in almost all Ising models with competing dynamics, we have found a particular case in which the stationary distribution can be expressed as a Gibbsian one with an effective Hamiltonian. Its coupling constants depend on the original parameters of the model. Because of this dependence, the macroscopic behavior of this model is highly nontrivial, as we have pointed out, for some interesting impure magnetic systems.

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