Modeling ionic diffusion in magnetic systems

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(Received 23 January 1998)

We present a solvable kinetic lattice model which includes microscopic disorder, namely, random exchange energies (between pairs of spins) fluctuating randomly with time. This, which may ideally model fast ionic diffusion in magnetic systems, impedes reaching equilibrium, in general. The resulting nonequilibrium steady states exhibit additional randomness which induces complex behavior, including reentrance phenomena reminiscent of spin-glass behavior. [S0163-1829(98)02738-6]

I. INTRODUCTION

The Ising *spin glass*[1,2] is a conceptually simple example of systems with microscopic disorder. It consists of the d dimensional (e.g.) sc lattice, Z^d, and the Hamiltonian

$$H_{\mathbf{J}}(\mathbf{s}) = -\sum_{\mathbf{x},\mathbf{y}\in Z^d} J_{\mathbf{xy}} s_{\mathbf{x}} s_{\mathbf{y}} - \sum_{\mathbf{x}\in Z^d} h_{\mathbf{x}} s_{\mathbf{x}}, \tag{1}$$

where $\mathbf{J}=\{J_{\mathbf{xy}}\in \mathrm{Re}; \mathbf{x},\mathbf{y}\in Z^d\}$ is a set of random numbers with given probability distribution. That is, each binary-spin configuration, $\mathbf{s}\equiv\{s_{\mathbf{x}}=\pm 1; \mathbf{x}\in Z^d\}$, has the potential energy (1) for each realization $\mathbf{J}$ of disorder. The fact that the coupling strength changes at random from one pair of spins to the other simulates that, in a class of alloys, magnetically active ions are spatially distributed at random, and exchange energies have a damped oscillatory behavior with distance. (Assuming random local fields, $h_{\mathbf{x}}$, in Eq. (1) is also interesting;[3–6] for simplicity, however, we restrict ourselves to constant external fields in this paper, i.e., $h_{\mathbf{x}}=h_e \ \forall \mathbf{x}$.) The set $\mathbf{J}$ is constant with time for each realization of the system, which corresponds to *quenched* disorder or frozen-in impurities. The interest is then on averages over both the stationary distribution for the configurations, $P_{\mathrm{st}}(\mathbf{s})$, and the weight, $p(\mathbf{J})$, of different realizations of disorder.[7–10]

The above has been generalized to include spatial distribution of disorder varying with time.[11,12] It has been argued that in order to model somewhat more realistically microscopically disordered magnetic systems, one needs to invoke time variations of the $J_{\mathbf{xy}}$s to take into account the consequences of ionic diffusion. This constantly modifies the distance $|\mathbf{x}-\mathbf{y}|$ between each specific pair of spins and, therefore, makes $J_{\mathbf{xy}}$ vary in practice also with time. As a first step towards understanding this effect, fast random diffusion of impurities has been studied. That is, one assumes that the probability of $\mathbf{s}$ at time t, $P_t(\mathbf{s})$, satisfies the master equation:

$$\frac{\partial P_t(\mathbf{s})}{\partial t} = \sum_{\mathbf{x}\in Z^d} [c(\mathbf{s}^{\mathbf{x}};\mathbf{x}) P_t(\mathbf{s}^{\mathbf{x}}) - c(\mathbf{s};\mathbf{x}) P_t(\mathbf{s})], \tag{2}$$

where the *rate* or probability per unit time for the transition (*flip*) from the configuration $\mathbf{s}\equiv\{\ldots, s_{\mathbf{x}-\mathbf{1}}, s_{\mathbf{x}}, s_{\mathbf{x}+\mathbf{1}}, \ldots\}$ to $\mathbf{s}^{\mathbf{x}}\equiv\{\ldots, s_{\mathbf{x}-\mathbf{1}}, -s_{\mathbf{x}}, s_{\mathbf{x}+\mathbf{1}}, \ldots\}$ is a superposition of more elementary processes, namely,

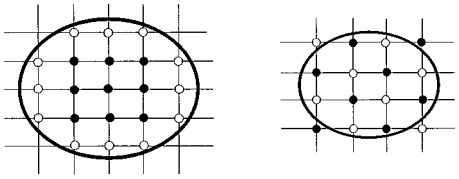
$$c(\mathbf{s};\mathbf{x}) = \int_{-\infty}^{+\infty} d\mathbf{J}\, p(\mathbf{J}) c_{\mathbf{J}}(\mathbf{s};\mathbf{x}). \tag{3}$$

The motivation for this is that, if the time variation of the $J_{\mathbf{xy}}$s (*diffusion*) is fast enough compared with spin changes, one may assume these evolving by a stochastic process characterized by an *effective*, competing rate as in Eq. (3).[12] The elementary rate $c_{\mathbf{J}}(\mathbf{s};\mathbf{x})$ concerns a particular distribution of $J_{\mathbf{xy}}$s and, therefore, a particular value of the energy $H_{\mathbf{J}}(\mathbf{s})$. For simplicity, it turns out convenient to deal in practice with functions $c_{\mathbf{J}}(\mathbf{s};\mathbf{x})$ that satisfy detailed balance, i.e., $c_{\mathbf{J}}(\mathbf{s};\mathbf{x}) = c_{\mathbf{J}}(\mathbf{s}^{\mathbf{x}};\mathbf{x})\exp[\beta H_{\mathbf{J}}(\mathbf{s}^{\mathbf{x}}) - \beta H_{\mathbf{J}}(\mathbf{s})]$. This implies that the system evolves with time towards the equilibrium (Gibbs) state corresponding to (inverse) temperature $\beta=(k_B T)^{-1}$ and energy $H_{\mathbf{J}}(\mathbf{s})$ if $p(\mathbf{J})=\Pi_{\mathbf{xy}}\delta(J_{\mathbf{xy}}-J)$, i.e., when the dynamics is due to the action of a unique $c_{\mathbf{J}}(\mathbf{s};\mathbf{x})$ alone. However, Eq. (3) describes more generally a competition of such tendencies whose net asymptotic result is a nonequilibrium steady state. Therefore, studying this system may be relevant to better understanding both ''impure'' systems and nonequilibrium phenomena.

We report in this paper on solvable versions of Eqs. (2) and (3), and compare with other approaches, namely, a pair approximation[13] and a computer simulation,[14] both concerning simpler cases. The situations we have investigated differ conceptually from the (rather unrealistic) annealed spin glass[15] in which impurities are in equilibrium with the spins —instead of constantly impeding canonical equilibrium. In fact, our study reveals certain behavior: the existence of the additional randomness implied by Eq. (3) amounts to a sort of dynamic frustration or conflict during the evolution which induces interesting complex phenomena.

II. DEFINITION OF MODEL

The explicit study of the general system (2),(3) is difficult,[12] so that we analyze in the following a simplified version of it. Our modification consists of a mean-field limit, since a large set of the system degrees of freedom is replaced by the action of a coherent field. That is, we are only concerned with details, including spin correlations, in a relatively small compact domain of lattice sites, Λ_C. This consists of two subsets, Λ_I and Λ_F, such that $\Lambda_C = \Lambda_I \cup \Lambda_F$, $\Lambda_I \cap \Lambda_F = 0$, Λ_I is the domain interior, and Λ_F is the *border*, defined as the subset of all sites of Λ_C that



(a) (b)

FIG. 1. (a) This illustrates the generic domain, $\Lambda_C \equiv \{\bullet, \circ\}$; the domain interior, $\Lambda_I \equiv \{\bullet\}$; and the *border*, $\Lambda_F \equiv \{\circ\}$. (b) A specific domain for $d=2$, consisting of 12 spins, as considered in the text; a distinction is made here between two sublattices, $\Lambda_A \equiv \{\bullet\}$, and $\Lambda_B \equiv \{\circ\}$.

have at least one nearest-neighbor outside Λ_C; cf. Fig. 1(a). The probabilities for the configurations, $\mathbf{s}_C = \{s_\mathbf{x}; \mathbf{x} \in \Lambda_C\}$, $\mathbf{s}_I = \{s_\mathbf{x}; \mathbf{x} \in \Lambda_I\}$, and $\mathbf{s}_F = \{s_\mathbf{x}; \mathbf{x} \in \Lambda_F\}$ may be related to each other by the equation,

$$P_t(\mathbf{s}_C) = P_t(\mathbf{s}_I/\mathbf{s}_F) P_t(\mathbf{s}_F), \tag{4}$$

where $P_t(\mathbf{s}_I/\mathbf{s}_F)$ is a conditional probability, and one assumes the normalizations $\Sigma_{\{\mathbf{s}_I\}} P_t(\mathbf{s}_I/\mathbf{s}_F) = 1$, and $\Sigma_{\{\mathbf{s}_F\}} P_t(\mathbf{s}_F) = 1$. A main feature of the model is that, for any particular configuration $\mathbf{s}_F$, $P_t(\mathbf{s}_I/\mathbf{s}_F)$ satisfies the master equation:

$$\frac{\partial P_t(\mathbf{s}_I/\mathbf{s}_F)}{\partial t} = \sum_{\mathbf{x} \in \Lambda_I} [c(\mathbf{s}_I^\mathbf{x}; \mathbf{x}) P_t(\mathbf{s}_I^\mathbf{x}/\mathbf{s}_F) - c(\mathbf{s}_I; \mathbf{x}) P_t(\mathbf{s}_I/\mathbf{s}_F)], \tag{5}$$

where $c(\mathbf{s}_I; \mathbf{x})$ is a superposition similar to Eq. (3), namely,

$$c(\mathbf{s}_I; \mathbf{x}) = [[c_\mathbf{J}(\mathbf{s}_I; \mathbf{x})]] \equiv \int_{-\infty}^{+\infty} d\mathbf{J}\, p(\mathbf{J}) c_\mathbf{J}(\mathbf{s}_I; \mathbf{x}). \tag{6}$$

The above is complemented with a choice for $P_t(\mathbf{s}_F)$, which involves an appropriate coherent field to take into account the relation between the spins in Λ_F and those in the rest of the lattice. That is, the total probability $P_t(\mathbf{s})$ in the original system is replaced here by the domain probability $P_t(\mathbf{s}_C)$ which invokes a field to be estimated self-consistently. This produces a set of 2^{N_I} coupled equations (instead of 2^N, $N \gg N_I \equiv \mathrm{card}(\Lambda_I)$, in the original problem) whose solution is feasible for not too large N_I.

The steady state is defined as the solution of $\partial P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F)/\partial t = 0$. One may write quite generally that $P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F) \equiv \exp\{-\beta \mathcal{H}(\mathbf{s}_I, \mathbf{s}_F) - \Gamma(\mathbf{s}_F)\}$, which defines $\mathcal{H}(\mathbf{s}_I, \mathbf{s}_F)$, where $\Gamma(\mathbf{s}_F)$ is determined by the normalization of $P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F)$. It follows that

$$\mathcal{H}(\mathbf{s}_I, \mathbf{s}_F) = \frac{1}{2^{N_I}} \ln \left[\prod_{\{\mathbf{s}_I\}} P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F) \right] - \ln P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F), \tag{7}$$

where we have assumed that $\mathcal{H}(\mathbf{s}_I, \mathbf{s}_F)$ can be written as a combination of products of spins inside the domain, i.e.,

$$\mathcal{H}(\mathbf{s}_I, \mathbf{s}_F) = \sum_{k=1}^{N_I} \sum_{(\mathbf{j}_1 \cdots \mathbf{j}_k)} J^{(k)}_{\mathbf{j}_1 \cdots \mathbf{j}_k} s_{\mathbf{j}_1} \cdots s_{\mathbf{j}_k}, \tag{8}$$

where $J^{(k)}_{\mathbf{j}_1 \cdots \mathbf{j}_k}$ are real numbers. Let us define the domain configurational energy as

$$H_\mathbf{J}(\mathbf{s}_C) = -\sum_{\mathbf{x},\mathbf{y} \in \Lambda_C} J_{\mathbf{x}\mathbf{y}} s_\mathbf{x} s_\mathbf{y} - h_e \sum_{\mathbf{x} \in \Lambda_C} s_\mathbf{x}. \tag{9}$$

If we write the probability of the border as

$$P_{\mathrm{st}}(\mathbf{s}_F) \propto \exp\left\{ \Gamma(\mathbf{s}_F) + \beta h \sum_{\mathbf{y} \in \Lambda_F} s_\mathbf{y} \right\}, \tag{10}$$

where $\Gamma(\mathbf{s}_F)$ is the same function as before, one obtains after using Eq. (4) that

$$P_{\mathrm{st}}(\mathbf{s}_C) = \frac{1}{\mathcal{N}} \exp\left\{ -\beta \mathcal{H}(\mathbf{s}_I, \mathbf{s}_F) + \beta h \sum_{\mathbf{y} \in \Lambda_F} s_\mathbf{y} \right\}, \tag{11}$$

with

$$\mathcal{N} = \sum_{\{\mathbf{s}_C\}} \exp\left\{ -\beta \mathcal{H}(\mathbf{s}_I, \mathbf{s}_F) + \beta h \sum_{\mathbf{y} \in \Lambda_F} s_\mathbf{y} \right\}. \tag{12}$$

The field h on the border is to be estimated self-consistently, i.e., by requiring the same magnetization per site in Λ_I than in Λ_F. One may easily generalize this, for example, by involving several different fields if required by the nature of order, as made explicitly in the next section.

III. SOME CASES OF THE MODEL

The formalism in the previous section is valid for any dimension d and size of Λ_I. We next deal with some cases for which the model has a simple analytical solution exhibiting nontrivial behavior. More complex cases may be studied by solving implicit equations in the computer.

Consider first the simplest case, namely, $\Lambda_I = \{x_0\}$ (any d) so that the domain consists of an interior (central) spin, $s_{\mathbf{x}_0}$, and its q nearest neighbors. For simplicity, we restrict ourselves in the following to elementary rates such that $c_\mathbf{J}(\mathbf{s}_I; \mathbf{x}) = \phi(\beta \Delta H^\mathbf{x}_\mathbf{J}(\mathbf{s}_C))$, where (using an obvious notation) $\Delta H^\mathbf{x}_\mathbf{J}(\mathbf{s}_C) \equiv H_\mathbf{J}(\mathbf{s}_I^\mathbf{x}, \mathbf{s}_F) - H_\mathbf{J}(\mathbf{s}_I, \mathbf{s}_F)$, and one has $\phi(X) = \phi(-X)\exp(-X)$—which corresponds to the detailed balance condition—, $\phi(0) = 1$, and $\phi(\infty) = 0$. Three familiar choices consistent with these properties are $\phi(X) = \exp(-X/2)$ (hereafter referred to as *rate A*), $\phi(X) = 2(1 + e^X)$ (*rate B*), and $\phi(X) = \min\{1, e^{-X}\}$ (*rate C*). For the indicated domain, the stationary conditional probability follows from $\partial P_{\mathrm{st}}(\mathbf{s}_I/\mathbf{s}_F)/\partial t = 0$ as

$$P_{\mathrm{st}}(s_{\mathbf{x}_0}/\mathbf{s}_F) = \frac{c(s_{\mathbf{x}_0}; \mathbf{x}_0)}{c(s_{\mathbf{x}_0}; \mathbf{x}_0) + c(-s_{\mathbf{x}_0}; \mathbf{x}_0)} \tag{13}$$

after using Eqs. (4) and (11). One may check that the condition $P_{\mathrm{st}}(-s_{\mathbf{x}_0}/\mathbf{s}_F)/P_{\mathrm{st}}(s_{\mathbf{x}_0}/\mathbf{s}_F) = c(-s_{\mathbf{x}_0}; \mathbf{x}_0)/c(s_{\mathbf{x}_0}; \mathbf{x}_0)$ holds, i.e., this is a very simple case in which the resulting effective rate—not only the elementary one—also satisfies detailed balance. One obtains from Eq. (7) that

$$\beta \mathcal{H}(s_{\mathbf{x}_0}, \mathbf{s}_F) = -\frac{1}{2} s_{\mathbf{x}_0} \ln \frac{[[\phi\{\beta \Delta^{\mathbf{x}_0}_\mathbf{J}(\mathbf{s}_F)\}]]}{[[\phi\{-\beta \Delta^{\mathbf{x}_0}_\mathbf{J}(\mathbf{s}_F)\}]]} \tag{14}$$

with $\Delta^{\mathbf{x}_0}_\mathbf{J}(\mathbf{s}_F) = 2\{\Sigma_{\mathbf{y} \in \Lambda_F} J_{\mathbf{x}_0 \mathbf{y}} s_\mathbf{y} + h_e\}$. Therefore,

$$\mathcal{H}(s_{\mathbf{x}_0},\mathbf{s}_F)=-\sum_{\mathbf{y}\in\Lambda_F} J_{\mathbf{x}_0\mathbf{y}}s_{\mathbf{y}}s_{\mathbf{x}_0}, \tag{15}$$

where

$$\beta\sum_{\mathbf{y}\in\Lambda_F} J_{\mathbf{x}_0\mathbf{y}}s_{\mathbf{y}}=-\frac{1}{2}\ln\frac{[[\phi\{\beta\Delta_{\mathbf{J}}^{\mathbf{x}_0}(\mathbf{s}_F)\}]]}{[[\phi\{\beta\Delta_{\mathbf{J}}^{\mathbf{x}_0}(\mathbf{s}_F)\}]]}. \tag{16}$$

This is an Ising-like *effective Hamiltonian*[16] to be associated to the domain of interest.

The restriction of these equations to $\Lambda_I=\{x_0\}$ (which makes them quite inappropriate to describe relatively complex, e.g., antiferromagnetic order) has compelled us to invoke a unique coherent field, h, in Eq. (10). The final solution is thus Eq. (11) with Eq. (14), where h is given by the self-consistence relation

$$\langle s_{\mathbf{x}_0}\rangle=\left\langle\frac{1}{q}\sum_{\mathbf{y}\in\Lambda_F} s_{\mathbf{y}}\right\rangle \tag{17}$$

with $\langle\cdots\rangle$ representing the average with respect to $P_{\rm st}(\mathbf{s}_C)$. This may be transformed into

$$\sum_{n=0}^{q}\binom{q}{n}y^n\Theta(n)=0, \tag{18}$$

where $y=e^{2\beta h}$, and

$$\Theta(n)=2n\left[\left[\frac{A(n)}{B(n)}\right]^{1/2}+\left[\frac{B(n)}{A(n)}\right]^{1/2}\right]-2q\left[\frac{B(n)}{A(n)}\right]^{1/2}, \tag{19}$$

with $A(n)=[[\phi(2\beta J(2n-q)+2\beta h_e)]]$ and $B(n)=[[\phi(-2\beta J(2n-q)-2\beta h_e)]]$. Before analyzing the consequences of Eq. (18), we apply the formalism in Sec. II to a larger domain.

Consider $d=2$ and Λ_C as defined in Fig. 1(b), i.e., the domain contains four interior spins surrounded by eight border spins. Let us define two sublattices, Λ_A and Λ_B, with $\Lambda_C=\Lambda_A\cup\Lambda_B$, $\Lambda_A\cap\Lambda_B=0$, as indicated in the figure, and the order parameters

$$m=\frac{1}{N}\sum_{\mathbf{x}\in\Lambda} s_{\mathbf{x}},\quad \mu=\frac{1}{2N_A}\sum_{\mathbf{x}\in\Lambda_A} s_{\mathbf{x}}-\frac{1}{2N_B}\sum_{\mathbf{x}\in\Lambda_B} s_{\mathbf{x}}. \tag{20}$$

It turns out necessary to invoke two coherent fields, h_A and h_B, acting on the border spins in Λ_A and Λ_B, respectively. That is, the domain probability is now

$$P_{\rm st}(\mathbf{s}_C)=\frac{1}{\mathcal{N}}\exp\left\{-\beta\mathcal{H}(\mathbf{s}_I,\mathbf{s}_F)+\beta h_A\sum_{\mathbf{y}\in\Lambda_{F_A}} s_{\mathbf{y}}+\beta h_B\sum_{\mathbf{y}\in\Lambda_{F_B}} s_{\mathbf{y}}\right\} \tag{21}$$

with

$$\mathcal{N}=\sum_{\{\mathbf{s}_C\}}\exp\left\{-\beta\mathcal{H}(\mathbf{s}_I,\mathbf{s}_F)+\beta h_A\sum_{\mathbf{y}\in\Lambda_{F_A}} s_{\mathbf{y}}+\beta h_B\sum_{\mathbf{y}\in\Lambda_{F_B}} s_{\mathbf{y}}\right\}. \tag{22}$$

The corresponding self-consistence relations are

$$2\left\langle\sum_{\mathbf{x}\in\Lambda_I} s_{\mathbf{x}}\right\rangle=\left\langle\sum_{\mathbf{x}\in\Lambda_F} s_{\mathbf{x}}\right\rangle,$$

$$2\left\langle\sum_{\mathbf{x}\in\Lambda_{I_A}} s_{\mathbf{x}}-\sum_{\mathbf{x}\in\Lambda_{I_B}} s_{\mathbf{x}}\right\rangle=\left\langle\sum_{\mathbf{x}\in\Lambda_{F_A}} s_{\mathbf{x}}-\sum_{\mathbf{x}\in\Lambda_{F_B}} s_{\mathbf{x}}\right\rangle. \tag{23}$$

One may write this squematically as

$$g_1(x,y)=0,\quad g_2(x,y)=0, \tag{24}$$

where $x\equiv e^{2\beta h_A}$, $y\equiv e^{2\beta h_B}$, and the functions g_j, $j=1,2$, depend on the choice for $p(\mathbf{J})$.

IV. RESULTS

We describe in this section some of the consequences of the equations in the previous one. Concerning the simplest case $\Lambda_I=\{x_0\}$, the solution is Eq. (11) with Eqs. (14) and (18). For $h_e=0$, the latter has the symmetry $\Theta(n)=-\Theta(q-n)$. It then follows that $y=1$ always satisfies Eq. (18), which corresponds to the trivial solution $m=0$. We next investigate the possibility of spontaneous magnetization, i.e., nonzero solutions for the domain $\Lambda_I=\{x_0\}$.

In the absence of competition, namely, for $p(\mathbf{J})=\Pi_{\mathbf{xy}}\delta(J_{\mathbf{xy}}-J_0)$, one has that $P_{\rm st}(\mathbf{s}_C)=Z_{\rm BP}^{-1}\exp\{-\beta H_{\rm BP}(\mathbf{s}_C)\}$, where $H_{\rm BP}(\mathbf{s}_C)=-J_0\Sigma_{\mathbf{y}\in\Lambda_F}s_{\mathbf{x}_0}s_{\mathbf{y}}-h\Sigma_{\mathbf{y}\in\Lambda_F}s_{\mathbf{y}}$ is the Bethe-Peierls Hamiltonian. That is, the system reproduces the familiar mean-field approximation for an equilibrium system. In fact, Eq. (18) reduces to

$$y=\left\{\frac{1+ye^{2\beta J_0}}{y+e^{2\beta J_0}}\right\}^{q-1}, \tag{25}$$

which has nonzero solutions for $T<T_c$, where $J_0/k_BT_c=\frac{1}{2}\ln(q/(q-2))$ locates the transition from a paramagnetic state to a (ferromagnetic) state with spontaneous magnetization.

Under competing dynamics for $d=2$, Eq. (18) reduces to $\Theta(0)y^2+4\Theta(1)y+\Theta(0)=0$ which has two nonzero solutions:

$$y_\pm=-\frac{2\Theta(1)}{\Theta(0)}\pm\sqrt{\left\{\frac{2\Theta(1)}{\Theta(0)}\right\}^2-1}. \tag{26}$$

These transform into a unique solution for $\Theta(1)/\Theta(0)=\frac{1}{2}$, which defines the critical temperature T_c for the second-order phase transition. The value for T_c depends on $p(\mathbf{J})$ and $\phi(X)$. As an illustration, we have explicitly worked out Gaussian and bimodal distributions for the disorder.

Consider first

$$p(\mathbf{J})=\prod_{\mathbf{xy}}\frac{1}{\sqrt{2\pi\tilde{J}}}\exp\left\{-\frac{(J_{\mathbf{xy}}-J_0)^2}{2\tilde{J}^2}\right\}. \tag{27}$$

For rate A, the phase transition is similar to the equilibrium one above; in particular, $k_BT_c/J_0=2.88$ for $d=2$, independent of the parameters in (27). That is, the dynamic competition does not induce any noticeable impure effect. For both rate B and rate C, by contrast, T_c depends on the width $\tilde{J}$. As $\tilde{J}/J_0\to0$, one observes that the zero-temperature magnetization, $m(T=0)$, tends to saturation; that is, the effect of

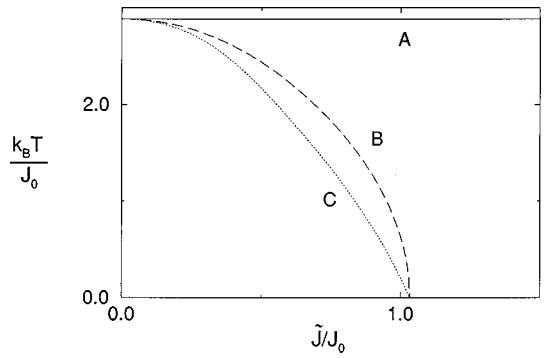


FIG. 2. The phase diagram for the two-dimensional system with the Gaussian dynamic competition (27) and (from top to bottom, as indicated) rates A, B, and C, as defined in Sec. III, when the relevant domain has a single interior spin.

impurities decreases as the Gaussian transforms into a δ function and, eventually, one recovers the equilibrium case with no competition. On the other hand, $m(T)\to 0$ for any T as $\tilde{J}/J_0\to 1.0336$, where the latter estimate is obtained as the solution of $\mathrm{erf}(J_0/\sqrt{2}\tilde{J})=\frac{2}{3}$. That is, a broad distribution of disorder makes so important the randomness that any long-range order is suppressed; interestingly, this occurs suddenly at a well-defined value of $\tilde{J}/J_0$. Figure 2 represents the phase diagram for various rates when the dynamic competition is Gaussian, Eq. (27). The fact that the rate so strongly influences the steady state is a distinguishing feature of kinetic systems with competing dynamics such as Eqs. (2)–(3).[12]

Consider next

$$p(\mathbf{J})=\prod_{\mathbf{xy}}\{p\,\delta(J_{\mathbf{xy}}-J_0)+(1-p)\,\delta(J_{\mathbf{xy}}+J_0)\}, \quad (28)$$

i.e., $J_{\mathbf{xy}}$ equals J_0 with probability p (both with time along the time evolution and spatially in any given configuration), and $-J_0$ with probability $1-p$. The steady state in this case is independent of the rate (for the family of rates considered), unlike for distribution (27). However, the probability p that the bond is positive plays a role similar to the width for the Gaussian distribution above. In particular, a transition of second order is exhibited at $T_c(p)$, such that saturation requires $p\to 1$, and no transition occurs for $p<p_0$. One obtains $p_0=\frac{5}{6}$ for $d=2$. Figure 3 illustrates the situation in this case,

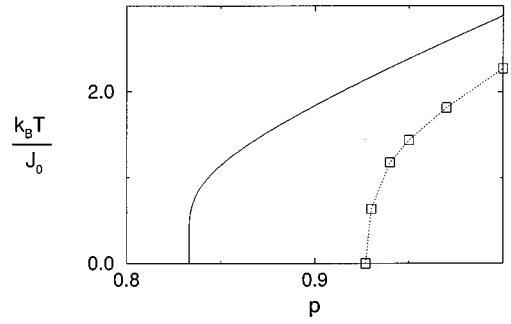


FIG. 3. The phase diagram for the two-dimensional system with the bimodal dynamic competition (28) when the relevant domain has a single interior spin; the result is independent of the elementary rate for rates A, B, and C. The symbols correspond to computer simulation results in Ref. 14 for the system (2),(3) with rate C.

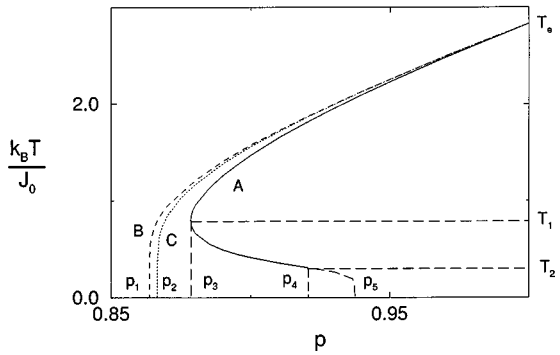


FIG. 4. The phase diagram for the two-dimensional domain in Fig. 1(b) and the bimodal dynamic competition (28). Different curves are for rates B, C, and A, from top to bottom, respectively.

and compares with data from a computer simulation of the system (2),(3) with rate C.[14]

The solution for the (two-dimensional) domain in Fig. 1(b) is Eq. (21) with Eq. (24). Let us consider now this case when the time evolution involves the bimodal distribution, i.e., Eq. (6) with Eq. (28). The functions in Eq. (24) for this distribution are

$$\begin{aligned} g_1(x,y)=&\,a_1(1-x^4y^4)+a_2(y+x-x^4y^3-y^4x^3)\\ &+a_3(y^2+x^2-x^4y^2-x^2y^4)\\ &+a_4(y^3+x^3-x^4y-xy^4)\\ &+a_5(xy-x^3y^3)+a_6(xy^2+x^2y-x^3y^2-x^2y^3), \end{aligned} \quad (29)$$

and

$$\begin{aligned} g_2(x,y)=&\,b_1(y-x-x^4y^3+x^3y^4)\\ &+b_2(y^2-x^2+x^2y^4-x^4y^2)\\ &+b_3(y^3-x^3+xy^4-x^4y)\\ &+b_4(y^4-x^4)+b_5(xy^2-x^2y-x^3y^2+x^2y^3)\\ &+b_6(xy^3-x^3y), \end{aligned} \quad (30)$$

where the coefficients, a_i and b_i, $i=1,\ldots,6$, depend on T, p, and on the function $\phi(X)$. These equations lead to the following principal results. For $p=1$ and 0, the critical temperature (for transitions from paramagnetic to ferromagnetic or antiferromagnetic phases, respectively) is $k_BT_c/J_0=2.8309$, slightly smaller than the corresponding Bethe value of 2.88 (cf. above); otherwise, this model version gives curves $m(T)$ that are very close to the ones for the Bethe-Peierls solution. For $p\in(0,1)$, the behavior strongly depends on the choice for $\phi(X)$. For rates B and C, T_c is a function of p; the function is different for each rate, and vanishes for $p<p_0$ with $p_0=0.8636$, 0.8664 for the cases B and C, respectively. The latter value is to be compared with $p_0=0.8649$ obtained from the pair approximation,[13] and $p_0=0.928$ obtained from computer simulation,[14] both for the same system. The phase diagram is presented in Fig. 4. Except for the fact that the rate determines here the steady state, the differences with the situation in Fig. 3—for rates B and C — are small. (Note that Fig. 4 is for $p>\frac{1}{2}$; the situation is

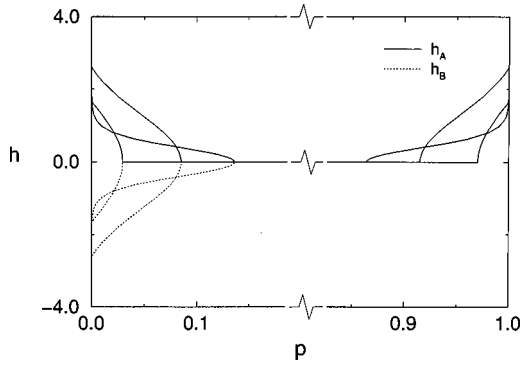


FIG. 5. Three illustrative isotherms showing the variation of the coherent field h with p for the system in Fig. 4 with rate B.

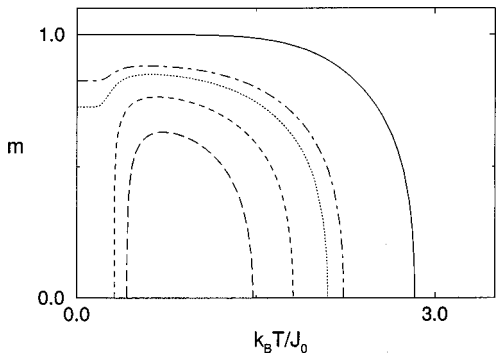


FIG. 6. Variation with temperature of the magnetization for different cases, for the domain in Fig. 1(b) and rate A. The curves are for the Bethe-Peierls (equilibrium) solution, and for the kinetic system with $p=0.95$, 0.94, 0.92, and 0.90, respectively, from top to bottom. The situation is similar for $p<\frac{1}{2}$ in the antiferromagnetic phase.

similar for the antiferromagnetic phase, $p<\frac{1}{2}$.) Figure 5 illustrates the variation of the coherent field h with p at three different temperatures (for rate B). This shows both the ferromagnetic and the antiferromagnetic regions; the latter exhibits the expected symmetry for the two involved fields.

The situation for rate A turns out to be quite different, as indicated in Fig. 4. The critical temperature, $T_c=T_1(p)$, locates, as before, a transition from paramagnetic to ferro or antiferromagnetic phases. However, when the temperature is below a second critical value, $T_2(p)$, another second-order phase transition occurs between the *ferromagnetic* phase and a phase that apparently (in the present description) does not exhibit any long-range order. We interpret that a new kind of order, different from the ordinary, ferromagnetic long-range order, occurs below $T_2(p)$ that characterizes a kind of *spin-glass* phase. As shown in Fig. 4, this *reentrant* phase transition occurs for $p\in[0.8782,0.9206]$, it becomes of first order for $p\in[0.9206,0.9375]$, and it does not occur for $p>0.9375$. Figure 6 depicts different magnetization curves confirming this picture.

Summing up, we studied a solvable mean-field version of a kinetic Ising model of disorder proposed earlier;[11] this extends our previous study of simpler cases.[13,14] The case presented here can systematically be generalized in the computer. A key feature of our system is that it allows for economical evaluation of the effects of ionic diffusion on the observed properties of spin-glass materials. It is assumed in Eqs. (2),(3) that ionic diffusion is so fast as compared to spin flips that one can take care of coupling changes by means of an effective, competing dynamics. In order to obtain an explicit case, we have approximated Eqs. (2),(3) by the coherent-field description in Sec. II whose steady state may be obtained exactly, e.g., Eqs. (21)–(24). Within this context, ionic diffusion has two principal effects. On one hand, it relieves some of the features, such as freezing phenomena that seem to characterize *spin-glass behavior*. On the other hand, diffusion makes the system relax to a *nonequilibrium* steady state, in general. Consequently, the details of the relaxation process—such as the rate in the model—determine the steady state. We have shown that, for some choices of parameters, the model exhibits interesting behavior, e.g., reentrance phenomena. Therefore, modeling appropriately the process of ionic diffusion, and perhaps including other effects (e.g., local random fields), one may induce complex model behavior. This strongly suggests that ionic diffusion in magnetic substances may be at the origin of some of the (often *peculiar*) observed properties of disordered systems.

ACKNOWLEDGMENTS

This work was supported by the DGICYT under Project No. PB91-0709 and by the *Junta de Andalucía*.

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