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Miguel Rubí
Conrado Pérez-Vicente (Eds.)

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**Editors**

Miguel Rubí
Conrado Pérez-Vicente
Dept. Física Fonamental
University of Barcelona
Diagonal 647
E-08028 Barcelona, Spain

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# A Kinetic Description of Disorder

P.L. Garrido[1], J. Marro[1] and M.A. Muñoz[1,2]

1 Instituto Carlos I de Física Teórica y Computacional. Facultad de Ciencias. Universidad de Granada. 18071-Granada, España.

2 T. J. Watson IBM Research Center, P. O. Box 218, Yorktown Heights, New York 10598, U.S.A.

**Abstract.** Some strategies to study the kinetics of disorder in Ising-like systems are reviewed.

## 1. Introduction

It has been known for a long time that the behavior of a physical system may drastically be affected by the presence of impurities. In order to get some insight into the role played by impurities, defects or, in general, *disorder*, many different models have been proposed in the context of equilibrium statistical mechanics. One of the most straightforward ways to implement disorder into well-known pure equilibrium models, consist on modifying its interaction Hamiltonian, in such a way that its translation invariance symmetry is broken. This can be achieved by introducing local variables in the Hamiltonian that take random values from site to site. Being more specific, let us consider, for example, a situation in which the translation invariant pure system is a $d$-dimensional lattice with sites occupied by spin variables. Each spin configuration, $\underline{s} \equiv \{s_{\underline{x}} = \pm 1; \underline{x} \in Z^d\}$, has an associated energy given by the Hamiltonian:

$$H(\underline{s}; \underline{J}) = -\sum_{A \subset Z^d} J_A s_A \quad , \quad s_A \equiv \prod_{\underline{x} \in A} s_{\underline{x}} \quad , \qquad (hamil)$$

where $\underline{J} = \{J_A | A \subset Z^d\}$. The translation invariance property reflects in the fact that the Hamiltonian has the symmetry: $T_{\underline{z}} H(\underline{s}; \underline{J}) = H(T_{\underline{z}} \underline{s}; \underline{J}) = H(\underline{s}; \underline{J}) \quad \forall \underline{z} \in Z^d$, where the operator $T_{\underline{z}}$ displaces any configuration by $\underline{z}$: $\underline{s'} \equiv T_{\underline{z}} \underline{s} = \{s'_{\underline{x}} = s_{\underline{x}-\underline{z}}; \underline{x} \in Z^d\}$. This property, if applied to the Hamiltonian (hamil) implies that $J_{T_{\underline{x}} A} = J_A \quad \forall \underline{x}, A$.

Let us assume first that the set $A$ in (hamil) consists of all possible different pairs of nearest-neighbor sites in a hypercubic lattice. The translation invariance property implies in this case that $J_A = J \quad \forall A$, and equation (hamil) reduces to the Ising Hamiltonian. On the other hand, when the translation invariance symmetry is broken, several models of disorder are recovered. For example:

(i) Assuming that, for every set $A$, $J_A$ is a random variable distributed around zero, we get the *Edwards-Anderson model* (Edwards and Anderson 1975). This exhibits *frustration* (the spatial competition between positive and negative couplings, $J_A$, prevents that all the exchange interactions in eq.(hamil) are simultaneously minimized) that induces *rare* macroscopic behavior observed in a class of materials known as *spin glasses*; see, for instance, the review book by Fischer and Hertz (1991).

(ii) If $J_A$ is a random variable taking only two different discrete values, $J$ or 0 with probabilities $p$ and $1-p$, respectively, we get the *bond dilute Ising model* which describes the physics of impure magnetic systems; see, for instance, the review by Stinchcombe (1983). These two models assume that impurities are quenched in the lattice, *i.e.* , their kinetics is neglected.

A simple way to induce some kinetics of disorder in Ising-like models, is to consider the couplings $J$ as thermal variables. In this case, there is time variation of the spatial distribution of $J$'s that is determined by the need to reach equilibrium with the spin degrees of freedom. This is the *anneal Ising model* (Thorpe and Beeman 1976). However, impurities tend in this case to be strongly correlated (for instance, located at the interfaces below the critical point) which is not observed in general.

The above (equilibrium) models and some variations of them exhibit a rich macroscopic behavior, and their study has helped the understanding of many phenomena first observed in real materials. Nevertheless, in order to study the broad variety of natural phenomena which appears to be related with disorder, a less restrictive scenario needs to be considered. In particular, many real systems under consideration are open to the environment, namely, driven by some external non-Hamiltonian force and/or interacting with external subsystems. Such general conditions may induce changes with time of the disorder variables. In practice, however, one needs to pay both analitically and

conceptually a high price for the consideration of kinetic disorder: steady *non-equilibrium* situations ensue due to added competition and extra randomness.

## 2. Definition of the basic model

We focus our attention here on the study of some non-equilibrium lattice models as defined via a master equation. Consider a system consisting of a lattice and spin, $\underline{s}$, and *disorder*, $\underline{J}$, degrees of freedom, which evolve stochastically according to a Markovian process (see, for instance, Garrido and Marro 1994). That is, the probability distribution, $\mu_t(\underline{s},\underline{J})$, that the system has at time $t$ the configuration $(\underline{s},\underline{J})$ satisfies the master equation:

$$\partial_t \mu_t(\underline{s},\underline{J}) = \left(L_{\underline{s}} + \Gamma L_{\underline{J}}\right) \mu_t(\underline{s},\underline{J}) \quad , \qquad (ME)$$

where

$$L_{\underline{s}} g(\underline{s},\underline{J}) = \sum_{\underline{x}} \left[c(\underline{s}^{\underline{x}} \to \underline{s}|\underline{J}) g(\underline{s}^{\underline{x}},\underline{J}) - c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J}) g(\underline{s},\underline{J})\right] \qquad (Ls)$$

$$L_{\underline{J}} g(\underline{s},\underline{J}) = \sum_{\underline{J}'} \left[w(\underline{J}' \to \underline{J}) g(\underline{s},\underline{J}') - w(\underline{J} \to \underline{J}') g(\underline{s},\underline{J})\right] \quad . \qquad (Lj)$$

Here, $g(\underline{s},\underline{J})$ stands for an arbitrary function, and $\underline{s}^{\underline{x}}$ is the configuration $\underline{s}$ with the spin at site $\underline{x}$ flipped, *i.e.* $s_{\underline{x}} \to -s_{\underline{x}}$. The Glauber operator, $L_{\underline{s}}$, describes stochastic spin flips with an associated transition probability per unit time (*rate*) $c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J})$ for given $\underline{J}$. $L_{\underline{J}}$ induces stochastic changes on $\underline{J}$ with rate $w(\underline{J} \to \underline{J}')$, and it is assumed to be independent of the spin configuration (further possibilities have been studied in Garrido and Marro 1994).

Both kinetic and stationary properties of the model depend in general on the rate. However, if dynamics of $\underline{J}$ is suppressed, which corresponds to the *quenched* case, $\Gamma = 0$, and the rates $c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J})$ in eq.(Ls) satisfy the *detailed balance property*, namely,

$$c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J}) = c(\underline{s}^{\underline{x}} \to \underline{s}|\underline{J}) \exp\left[-\Delta H_{\underline{x}}\right] , \Delta H_{\underline{x}} \equiv H(\underline{s}^{\underline{x}},\underline{J}) - H(\underline{s},\underline{J}), \quad (DB)$$

the stationary state corresponds to an equilibrium Gibbsian distribution, characterized by the Hamiltonian (hamil), *i.e.* , $\mu_{st}(\underline{s},\underline{J}) \propto \exp[-H(\underline{s},\underline{J})]$. Let us notice that, in this case, the detailed balance property (DB) implies

that the stationary state does not depend on the particular choice of $c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J})$. A simple, commonly considered choice, satisfying Eq. (DB), is

$$c(\underline{s} \to \underline{s}^{\underline{x}}|\underline{J}) = \Phi(\Delta H_{\underline{x}}) \qquad (rate)$$

where $\Phi$ is an arbitrary function with the property $\Phi(\lambda) = \Phi(-\lambda)\exp(-\lambda) \geq 0$. In particular, some specific examples are: $\Phi(\lambda) = 1 - \tanh(\lambda/2)$, $\min(1, \exp(-\lambda))$ and $\exp(-\lambda/2)$.

For $\Gamma > 0$, (ME) induces kinetics of the disorder degrees of freedom. In this case, the simultaneous action of the two kinetic mechanism in (ME), makes that, *a priori*, the steady state is not Gibbsian, and, contrary to what happens in the equilibrium, there is strong dependence on the analytical form of the dynamical mechanisms (rates).

There is not a general theory, analogous to the equilibrium ensembles theory, which relates in a simple and systematic way the microscopic structure of non-equilibrium systems and their macroscopic properties. The lack of a general theory makes the theoretical analysis of such systems to be quite complex, and usually approximate schemes are required to get some insight into their macroscopic behavior.

Under this circumstances, there are two different alternative approaches to deal with this kind of systems. On one hand, sometimes it is possible to find the exact solution of the probabilistic model by using some kind of statistical analysis (specially in dimension $d = 1$) or mapping the system somehow into an equilibrium one with effective parameters.

On the other hand, approximate schemes can be used. In particular, mean-field type approximations can be constructed. The problem with that sort of approach is that it is mainly based on the uncontrolled truncations of local system correlations. It is common to use numerical simulations together with mean field approaches to test one each other in order to get a comprehensive understanding of the system-macroscopic-behavior . An alternative type of approach is based in the representation of the original discrete Master equation, in terms of some continuous stochastic or partial differential equation. For the latter equation further analytical methods are available.

In the next two section we discuss some recent results in the two previously mentioned directions: namely, the derivation of exact results, and the construction of continuous descriptions of microscopic non-equilibrium models.

## 3. Some exact results: models with effective Hamiltonian

Let us consider the limiting case: $\Gamma \to \infty$. This limit represent physical situations in which the time scale for the evolution of the impurities is much faster that the characteristic time scale for the spin evolution. In other words, during the interval elapsed between two consecutive spin-flip processes, the disorder degrees of freedom undergo enough changes to assure that their associated probability distribution, $p(\underline{J})$, reaches its steady state,

$$L_{\underline{J}} p_{st}(\underline{J}) = 0. \qquad (ste)$$

One can easily argue (see, for instance Garrido and Marro 1994), that in this case, the spin probability distribution, $\rho_t(\underline{s}) = \sum_{\underline{J}} \mu_t(\underline{s}, \underline{J})$, is solution of the effective Master equation

$$\partial_t \rho_t(\underline{s}) = L^{eff}_{\underline{s}} \rho_t(\underline{s}) \qquad (MEef)$$

where

$$L^{eff}_{\underline{s}} = \sum_{\underline{J}} p_{st}(\underline{J}) L_{\underline{s}} \qquad (Leff)$$

is an effective Glauber spin flip operator similar to $L_{\underline{s}}$ in eq. (Ls).

Assuming that $c(\underline{s} \to \underline{s}^{\underline{x}} | \underline{J})$ has the equilibrium analytical form given by equation (rate), the effective rate associated with the Glauber operator $L^{eff}_{\underline{s}}$ can be written as

$$c_{eff}(\underline{s} \to \underline{s}^{\underline{x}}) = \sum_{\underline{J}} p_{st}(\underline{J}) \Phi(\Delta H_{\underline{x}}(\underline{J})). \qquad (ceff)$$

That is, an effective dynamics has been defined which is an stochastic superposition of different mechanisms, $\Phi(\Delta H_{\underline{x}}(\underline{J}))$, weighted with $p_{st}(\underline{J})$. Each of the mechanisms, acting by itself, would drive the system to a different equilibrium state, and the competition of many of them introduces a kind of *dynamical frustration*, which drives the system in general to a non-equilibrium stationary state.

The exact stationary solution of the master equation (MEef) can be found in many cases by working out whether the effective spin flip rate (ceff) holds the detailed balance property (DB) with respect some *effective Hamiltonian* (Garrido and Marro 1989). The disorder distribution, the spatial dimension

and the structure of the competing Hamiltonian determine the existence or non-existence of such an effective Hamiltonian [3].

To explicitely show that influence, let us consider as an specific example, the reference Hamiltonian (that is, the Hamiltonian used to define the competing rates (ceff)) to be the Ising one,

$$H(\underline{s}, J) = -J \sum_{|\underline{x}-\underline{y}|=1} s_{\underline{x}} s_{\underline{y}} \qquad (IS)$$

where the coupling $J$ is a random variable with a given stationary distribution $p_{st}(J)$.

¿From the detailed balance property (DB) one can show in the one dimensional case that the stationary distribution associated to the effective dynamics, defined by (MEef) and (Leff), is a Gibbsian distribution with an Ising Hamiltonian, that is

$$\rho_{st}(\underline{s}) \propto \exp[-H(\underline{s}, J_{eff})] \qquad (roef)$$

where

$$J_{eff} = \frac{1}{4} \ln \left[ \frac{\ll \Phi(-4J) \gg}{\ll \Phi(4J) \gg} \right] \qquad (keff)$$

and $\ll g(J) \gg \equiv \sum_J p_{st}(J) g(J)$ stands for the average of a given function $g(J)$ over the $J$ stationary distribution.

In this way, the competition of different equilibrium rates [4] drives the system to a Gibbsian state with the same reference Ising Hamiltonian but with an effective coupling parameter. Such effective coupling depends on the analytical form of the competing rates and on the disorder stationary state as can be concluded from (keff).

This simple picture is specific of the one-dimensional model and cannot be extended to higher dimensions (there are, however, some Monte Carlo

[3] Notice that even in cases in which a the detailed balance property does not hold, it is possible to have an stationary Gibbsian measure. A well known example is given by the rate $c(\underline{s} \to \underline{s^x}) = \exp[-s_{\underline{x}}(s_{\underline{x}+\underline{i}}+s_{\underline{x}+\underline{j}})]$ defined on a two dimensional square lattice with periodic boundary conditions. $\underline{i}$ and $\underline{j}$ are the unit lattice vectors in the $X$ and $Y$ axis directions respectively. This rate has not the detailed balance property but the master equation stationary solution is a Gibbsian measure with Ising Hamiltonian $H = -\sum_{NN} s_{\underline{x}} s_{\underline{y}}$.

[4] In the sense that each one acting alone drive the system to a Gibbsian state characterized by the one dimensional Ising Hamiltonian with *different* coupling.

computer simulations performed in the two-dimensional version of the previous model which show that the stationary solution is an equilibrium one with effective parameters, except at low temperatures (González-Miranda et al. 1994)).

To show the lack of robustness of the effective detailed balance property, even in one dimension, we point out that, there is no effective-Hamiltonian description when a fixed magnetic field is included in the Ising Hamiltonian (IS).

Nevertheless, there exists one particular case in which we are able to find an effective Hamiltonian for any dimension. It corresponds to: *1)* the particular function $\Phi(\lambda) = \exp(-\lambda/2)$ is considered in (ceff) *2)* the couplings $J_A$ of the Hamiltonian (hamil) are stochastically independent variables, which evolve uncorrelately. In other words, $p_{st}(\underline{J}) = \prod_{A \subset Z^d} p_A(J_A)$. The effective rate (ceff) is then written

$$c_{eff}(\underline{s} \to \underline{s}^{\underline{x}}) = \prod_{B \bigcap \{\underline{x}\}} \left[ \sum_{J_B} p_B(J_B) \exp\left(-J_B s_B\right) \right] \qquad (dd1)$$

In this situation one may show that the stationary state in any dimension is a Gibbsian one $\rho_{st}(\underline{s}) \propto \exp(-H(\underline{s}; \underline{J^{eff}}))$ (see for instance reference Garrido and Muñoz 1993), where

$$J_A^{eff} = \frac{1}{2} \ln \left[ \frac{\ll \exp(J_A) \gg}{\ll \exp(-J_A) \gg} \right] \qquad (JAeff)$$

Let us emphasize that we are not able to find an effective Hamiltonian when another analytical form of $\Phi$ is considered in (ceff), or if it is assumed that $J_A$ are correlated variables.

As a particular realization of equation (JAeff) let us choose the reference Hamiltonian to be the general d-dimensional Ising Hamiltonian,

$$H(\underline{s}, \underline{J}) = - \sum_{|\underline{x}-\underline{y}|=1} J_{\underline{x}\underline{y}} s_{\underline{x}} s_{\underline{y}} \qquad (GIM)$$

where now the set $B$ in equation (dd1) denotes pairs of nearest-neighbor sites in the lattice. For any disorder distribution of the form

$$p_{\underline{x}\underline{y}}(J) = f(J - J^0_{\underline{x}\underline{y}}) \qquad (dist)$$

with $f(J)$ being any probability distribution symmetric around zero, and $\{J^0_{\underline{x}\underline{y}}\}$ being a given set of couplings, it is straightforward to show that an effective Ising Hamiltonian exits with effective couplings given by: $J^{eff}_{\underline{x}\underline{y}} = J^0_{\underline{x}\underline{y}}$ *independently of any parameter of the distribution* $f$ (for instance, if $f$ is a Gaussian distribution with variance $\sigma$, the result will be $\sigma$-independent).

Other similar realizations of (JAeff) can easily be worked out (Garrido and Muñoz 1993). For instance, the *non-equilibrium impure Ising model* corresponding to $p_{\underline{x}\underline{y}}(J) = p\delta(J - J^0) + (1-p)\delta(J)$, or the *non-equilibrium spin glass Ising model* defined by $p_{\underline{x}\underline{y}}(J) = p\delta(J - J^0) + (1-p)\delta(J + J^0)$.

Let us finally mention a case in which we relax the aforementioned property, *2)*, in which we assumed that the disorder was totally uncorrelated, and we still have an effective Hamiltonian. This occurs in the so-called *kinetic ANNNI* model (López-Lacomba and Marro 1994). In this case, the Hamiltonian that defines the effective rate (dd1) is

$$H(\underline{s}; J, J') = -J \sum_{|\underline{x}-\underline{y}|=1} s_{\underline{x}} s_{\underline{y}} - J' \sum_{\underline{x}} s_{\underline{x}} s_{\underline{x}+2\underline{z}} \qquad (ANNI)$$

where $\underline{z}$ is the unit vector pointing to one of the lattice directions. Namely, the model is defined on a d-dimensional Ising model in which a next-nearest-neighbor interaction with strength $J'$ is added in one of the lattice directions. It is assumed that the coupling $J$ is fixed and $J'$ is a random variable, that is, $p_{st}$ depends only on $J'$. The effective Hamiltonian is found to be $H(\underline{s}; J, J'_{eff})$ with

$$J'_{eff} = \frac{1}{2} \ln \left[ \frac{\ll \exp(J') \gg}{\ll \exp(-J') \gg} \right] \qquad (Jpe)$$

All these models have interesting macroscopic behaviors which have been studied in the above commented references where we refer the reader for more details.

## 4. The continuum description: looking for a suitable starting point

This is a more broadly applicable method. It is suitable for the study of phase transitions and critical phenomena appearing in non-equilibrium Master equations. It is based on the observation that the strategies used in the study of the critical dynamic properties of systems evolving towards an equilibrium state can be extended to the analysis of non-equilibrium dynamical models. The idea is to construct a continuum version of the lattice model whose equation of motion is a stochastic differential equation (Langevin equation) [5].

The Langevin equation is a simplified representation of the microscopic system that contains the most relevant features to describe properties that depend on large scales in space and time. As the nature of critical phenomena is usually determined only by large-scale properties and not by specific microscopic details of the models, this approach is a natural way to study phase transitions in both equilibrium and non-equilibrium situations.

In principle, such continuum description should be derived from the microscopic Master Equation by means of a coarse-graining procedure. That is, changing in the original Master equation the spin variables by some new local variables defined as averages of the spins over a large region. One expects that, in an adequate limit, the microscopic details are averaged away and in the resulting Langevin description only the large scale properties remain. However, the coarse-graining procedure cannot be, in general, applied successfully without introducing some extra assumptions [6]. Nevertheless, for systems evolving towards an equilibrium distribution, it is used a very simple method to construct a Langevin equation. First, one knows that the system stationary distribution is given by a Gibbsian measure, $\rho \propto \exp[-H]$ for a

[5] We indistinctly will use the Langevin or the Fokker-Planck equations. Both are stochastically equivalent. The first one is a stochastic differential equation where the unknowns are the local fields. The Fokker-Planck is a second order functional differential equation in which the unknown is the probability to find the systems at a given time with a given configuration.

[6] A. de Masi, P. Ferrari and J.L. Lebowitz managed to make in a rigorous manner a coarse-graining procedure in a reaction diffusion model (de Masi et al. 1985) in which the diffusion process is infinitely faster than the reaction one.

given interaction Hamiltonian $H$. And second, one expects that the critical phenomena will be independent on the particular analytical form of the dynamics. That is, one should write down a Langevin equation such that it is guaranteed that the stationary state is the one given by the Gibbsian measure. That is the strategy followed by Hohenberg and Halperin (1977) in their, so called, model A, in order to describe the critical dynamics of systems with a non-conserved order parameter. The equation defining the model A is:

$$\partial_t \varphi_t(\underline{r}) = -\lambda \frac{\delta H}{\delta \varphi_t(\underline{r})} + \eta_t(\underline{r}) \qquad (HH)$$

where $\varphi_t(\underline{r})$ is the coarse-graining density field at a given time $t$ on the point $\underline{r}$ of the continuum space, and $\eta$ is a stochastic white noise that reflects the fluctuations on the density field due to the action of the microscopic dynamics. Equation (HH) is the starting point for the study of large scale dynamic properties in equilibrium systems.

Coming back to our non-equilibrium problem, we have obviously the same aforementioned technical problems in the coarse-graining procedure. Moreover, any extra assumption may now change dramatically the system macroscopic behavior. Finally, we cannot apply the Hohenberg-Halperin strategy because *we don't know what is the measure corresponding to the stationary state*, which is coherent with the fact that such state depends on the microscopic dynamics. Nevertheless, it is possible to apply the idea of constructing a Langevin equation such that it is guaranteed, without solving the equations, that its stationary distribution is equal to the exact one, solution of the non-equilibrium Master equation, at least in some basic aspects. To achieve that, it is necessary to introduce a suitable continuum version of our Master Equation (MEef).

Let us introduce now the continuum version of the non-equilibrium models we are going to deal with. The system consists of a d-dimensional lattice where at each site there is a spin-like variable. We define at each point, $\underline{r} \in R^d$, a field variable, $\varphi(\underline{r}) \in R$, which is the averaged value of the spins on a region of volume $\Omega$ around $\underline{r}$. When $\Omega$ is large enough, $\varphi$ is assumed to be a continuous function on $\underline{r}$. The probability to find a field configuration, $\underline{\varphi} = \{\varphi(\underline{r}); \underline{r} \in R^d\}$, at time $t$, say $P_t^{\Omega}(\underline{\varphi})$, evolves according to a Markovian Master equation:

$$\partial_t P_t^{\Omega}(\underline{\varphi}) = \int_{R^d} d\underline{r} \int_R d\eta f(\eta) \Big[ w^{\Omega}(\underline{\varphi}^{\eta,\underline{r}} \to \underline{\varphi}) P_t^{\Omega}(\underline{\varphi}^{\eta,\underline{r}}) - w^{\Omega}(\underline{\varphi} \to \underline{\varphi}^{\eta,\underline{r}}) P_t^{\Omega}(\underline{\varphi}) \Big], \qquad (MME2)$$

where $f(\eta)$, an even and analytical real function around the origin, stands for the field increments distribution, $\underline{\varphi}^{\eta,\underline{r}} = \{\varphi(\underline{r}') + \frac{\eta}{\Omega}\delta_{\underline{r},\underline{r}'}, \underline{r}' \in R^d\}$, and $w^{\Omega}(\underline{\varphi} \to \underline{\varphi}')$ represents the transition probabilities per unit time. These are defined as in eq.(ceff):

$$w^{\Omega}(\underline{\varphi} \to \underline{\varphi}') = \int_{R^d} d\underline{K}\ p_{st}(\underline{K}) w^{\Omega}(\underline{\varphi} \to \underline{\varphi}'; \underline{K}), \qquad (rate2)$$

where $w^{\Omega}(\underline{\varphi} \to \underline{\varphi}'; \underline{K}) = \Phi[H^{\Omega}(\underline{\varphi}'; \underline{K}) - H^{\Omega}(\underline{\varphi}; \underline{K})]$, and now $H^{\Omega}(\underline{\varphi}; \underline{K}) = \Omega \int_{R^d} d\underline{r}\ h(\underline{\varphi}(\underline{r}); \underline{K})$ is a continuum interaction Hamiltonian which depends on the parameters $\underline{K} = \{K_i, i = 1, .., n\}$.

When we rescale the time variable $\tau = \Omega^{-1}t$, and we do the limit $\Omega \to \infty$, the solution of the Master Equation (MME2) is $P_\tau(\underline{\varphi}) = \delta(\underline{\varphi} - \underline{v}_\tau)$ where $\underline{v}_\tau$ is the solution of the so called *deterministic equation*:

$$\partial_\tau v_\tau(\underline{r}) = -\Xi_0^{exact}(\underline{v}_t(\underline{r})) \equiv \int_R d\eta f(\eta)\eta \ll \Phi(U_\eta(\underline{v}_t(\underline{r}); \underline{K})) \gg, \qquad (det2)$$

being $U_\eta(\underline{\varphi}(\underline{r}); \underline{K}) = \eta \frac{\delta \hat{H}(\underline{\varphi}; \underline{K})}{\delta \underline{\varphi}(\underline{r})}$, and $\ll A \gg \equiv \int_{R^n} d\underline{K}\ p_{st}(\underline{K}) A(\underline{K})$. In general, for large enough $\Omega$, the stationary probability distribution solution of the Master Equation (MME2) can be written as: $P_{st}^{\Omega}(\underline{\varphi}) \propto \exp[-V_{st}^{\Omega}(\underline{\varphi})]$ where $V_{st}^{\Omega}(\underline{\varphi}) = \Omega V_{0,st}(\underline{\varphi}) + V_{1,st}(\underline{\varphi}) + O(\Omega^{-1})$. The non-equilibrium potential $V_{st}^{\Omega}(\underline{\varphi})$ is expected to be continuous but not differentiable in some small regions in the phase space (see for instance Graham and Tel, 1984, 1985, 1986) and it can be shown that it is a Lyapunov function for the underlying deterministic dynamical system (Jauslin 1987).

In this conditions, the following theorem can be proven (Garrido and Muñoz 1995; Muñoz and Garrido 1994):

*Theorem: Let any Fokker-Planck equation*

$$\partial_t P_t^{\Omega}(\underline{\varphi}) = \frac{1}{\Omega} \int_{R^d} d\underline{r} \frac{\delta}{\delta \varphi(\underline{r})} \left[ \Xi_0(\underline{\varphi}(\underline{r})) + \frac{1}{\Omega} \Xi_1(\underline{\varphi}(\underline{r})) \frac{\delta}{\delta \varphi(\underline{r})} \right] P_t^{\Omega}(\underline{\varphi}), \qquad (FP)$$

*such that its coefficients have the form*

$$\Xi_0(\underline{\varphi}(\underline{r})) = \frac{1}{2}\left[D_-(\underline{\varphi}(\underline{r})) - D_+(\underline{\varphi}(\underline{r}))\right]$$
$$\Xi_1(\underline{\varphi}(\underline{r})) = \frac{D_-(\underline{\varphi}(\underline{r})) - D_+(\underline{\varphi}(\underline{r}))}{2\ln\left[\frac{D_-(\underline{\varphi}(\underline{r}))}{D_+(\underline{\varphi}(\underline{r}))}\right]} \qquad (Th3)$$

*where* $D_\eta(\underline{\varphi}(\underline{r})) = \ll \Phi(U_\eta(\underline{\varphi}(\underline{r}); \underline{K})) \gg$. *When* $f(\eta) = \frac{1}{2}\Big[\delta(\eta-1) + \delta(\eta+1)\Big]$, *then*

*i) It reproduces the exact deterministic dynamics given by eq.(det2).*

*ii) The* $V_{0,st}$ *and* $V_{1,st}$ *parts of its stationary solution almost coincide with the exact one in a suitable neighborhood of all spatially homogeneous deterministic solutions* $\underline{v}^*$.

Using this theorem, it is possible to find a Fokker-Planck type of description for the competing-dynamics models which represent exactly the original stationary distribution in some regions of phase space. These regions are very relevant because they determine the stationary critical behavior of the system. With this description as starting point it is feasible to perform suitable analysis of the critical properties by using well known methods developed for the study of Fokker-Planck and Langevin equations (Jansen et al 1976).

Summarizing, we have defined a general model that includes a kinetic mechanism to describe the disorder (impurities) time evolution. That induces the associated stationary probability distribution to be a non-equilibrium one. The theoretical tools available to study such non-equilibrium systems are scarce and not too powerful. Nevertheless we have shown that one can get interesting results in the particular limit in which the disorder evolves in a time scale much shorter that the spins. In such situation, and for a particular analytical form of the microscopic dynamics, we have shown that one can find the exact stationary distribution which is in fact a Gibbsian one characterized by an effective Hamiltonian. However, the latter particular results cannot be generalize for other rates and one should go to a much simpler mesoscopic continuous descriptions to get some valid information about the system behavior. In that context, we have shown that it is possible to explicitely write down a Fokker Planck equation whose stationary state coincides with the exact one in some relevant parts of the phase space.

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