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Kinetic Lattice Models of Disorder

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We study a class of stochastic Ising (or interacting particle) systems that exhibit a spatial distribution of impurities that change with time. It may model, for instance, steady nonequilibrium conditions of the kind that may be induced by diffusion in some disordered materials. Different assumptions for the degree of coupling between the spin and the impurity configurations are considered. Two interesting well-defined limits for impurities that behave autonomously are (i) the standard (i.e., quenched) bond-diluted, random-field, random-exchange, and spin-glass Ising models, and (ii) kinetic variations of these standard cases in which conflicting kinetics simulate fast and random diffusion of impurities. A generalization of the Mattis model with disorder that describes a crossover from the equilibrium case (i) to the nonequilibrium case (ii) and the microscopic structure of a generalized heat bath are explicitly worked out as specific realizations of our class of models. We sketch a simple classification of transition rates for the time evolution of the spin configuration based on the critical behavior that is exhibited by the models in case (ii). The latter are shown to have an exact solution for any lattice dimension for some special choice of rates.

KEY WORDS: Stochastic Ising systems; interacting particle systems; steady nonequilibrium states; disordered systems; spin glasses; conflicting kinetics; time relaxation; heat bath.

1. INTRODUCTION, AND DEFINITION OF BASIC MODEL

The present understanding of cooperative phenomena in nature relies significantly on the success of the Ising model to capture essential physics in complex systems. Many *pure* systems have been modeled as a (e.g.) simple-cubic d-dimensional lattice whose sites are occupied by spin (1/2)

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variables whose configurations, $\mathbf{s} \equiv \{s_{\mathbf{x}} = \pm 1; \mathbf{x} \in \mathbf{Z}^d\}$, have potential energy given by

$$H(s) = - \sum_{|x-y|=1} J_{xy} s_x s_y - \sum_x h_x s_x \tag{1.1}$$

where the first sum is over all pairs of nearest-neighbor (NN) sites. A heat bath at temperature T is involved by setting up either a canonical ensemble (e.g., ref. 1) or else a kinetic evolution consisting of stochastic changes of $\mathbf{s}$.[2-5] Several conceptually simple variations of this case have been devised to study *impure* systems (e.g., refs. 6–14). The *Edwards–Anderson model*,[10] where J_{xy} is a random variable distributed around zero, is based on (1.1) (with $h_x = 0$ for any $\mathbf{x}$). The spatial competition between positive and negative couplings may prevent all the exchange interactions in (1.1) being simultaneously locally minimized. Such a *frustration* induces uncommon macroscopic behavior that is believed to be shared, at least in part, by a class of materials known as *spin glasses*.[8,11-13] Frustration has a different origin in the so-called *random field model*,[14] where only the local field h_x in (1.1) varies randomly with $\mathbf{x}$.

We identify the latter (and similar) systems as *models of quenched disorder* (MQD) from now on. Their conceptual simplicity is deceptive. New concepts and techniques arising in the study of MQD have increased our understanding of many phenomena, but some problems remain. Exact, and even approximate relevant results remain scarce, the comparison between the behavior of models and real materials is not quite satisfactory, and some controversy on basic issues persists.[8,11-16] Further study is advised; in particular, it seems desirable to consider modifications to fit some cases in nature better. Concerning spin glasses, for example, the MQD neglect ion diffusion. Diffusion constantly modifies the distance between each specific pair of magnetically active ions in natural substances. Consequently, one should probably allow for variations in a model both in space and time of J_{xy}. The same argument may be applied to other cases of disorder, e.g., random fields. We remark that such diffusion has a too simple, unrealistic representation within the annealed version of (e.g.) the Edwards–Anderson model.[17] That is, the time variation of the spatial distribution of J's is then constrained by the need to reach equilibrium at temperature T with the other (spin) degrees of freedom. Therefore, impurities tend to be strongly correlated, e.g., located at interfaces, which is not observed in general. Instead, one may imagine that both the spin configuration $\mathbf{s}$ and the spatial distribution of impurities (i.e., of the sets $\{J_{xy}\}$, $\{h_x\}$, or $\{J_{xy}, h_x\}$, depending on the situation of interest) vary with time in such a way that the latter behave rather independently of $\mathbf{s}$, e.g., completely at random. A steady nonequilibrium condition may then occur

asymptotically. We recall that some observations in substances with disorder are consistent with the existence of nonequilibrium effects, e.g., a *peculiar* time dependence, and a strong influence of the details of the process on the steady state have been reported.[12,15,16]

This situation and a general concern for steady nonequilibrium states have motivated us to study stochastic Ising systems with *dynamical* disorder. A precise relation exists between the MQD and the new models, but, unlike the former, different versions of the latter either have an exact solution or are amenable to simple, e.g., mean-field and Monte Carlo treatments. They allow us to evaluate the influence of diffusion of disorder on thermodynamics, which happens to be interesting. Moreover, different limiting conditions for the new models represent a variety of *impure* situations besides the familiar MQD, including nonequilibrium magnetic systems with competing kinetics[18-22] and neural networks.[23] Our models are defined as follows.

Consider the Ising model with spin configuration $\mathbf{s}$; $\mathbf{S} \equiv \{\mathbf{s}\}$. Let $\boldsymbol{\zeta} = \{\zeta_i \in \mathbb{R}\}$ be a set of degrees of freedom not included in $\mathbf{s}$, e.g., $\{J_{xy}\}$, $\{h_x\}$, etc. The probability of a given *whole configuration* $(\mathbf{s}, \boldsymbol{\zeta})$ at time t, say $P_t(\mathbf{s}, \boldsymbol{\zeta})$, satisfies the master equation

$$\partial P_t(\mathbf{s}, \boldsymbol{\zeta})/\partial t = (L_s + \Gamma L_\zeta)\, P_t(\mathbf{s}, \boldsymbol{\zeta}) \tag{1.2}$$

where

$$L_s g(\mathbf{s}, \boldsymbol{\zeta}) = \sum_{s \in S} [c(\mathbf{s}^x \to \mathbf{s} \mid \boldsymbol{\zeta})\, g(\mathbf{s}^x, \boldsymbol{\zeta}) - c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\zeta})\, g(\mathbf{s}, \boldsymbol{\zeta})] \tag{1.3}$$

and

$$L_\zeta g(\mathbf{s}, \boldsymbol{\zeta}) = \sum_{\boldsymbol{\eta} \in \mathbf{H}} [w(\boldsymbol{\eta} \to \boldsymbol{\zeta} \mid \mathbf{s})\, g(\mathbf{s}, \boldsymbol{\eta}) - w(\boldsymbol{\zeta} \to \boldsymbol{\eta} \mid \mathbf{s})\, g(\mathbf{s}, \boldsymbol{\zeta})] \tag{1.4}$$

Here, g stands for an arbitrary function, $\mathbf{H} \equiv \{\boldsymbol{\eta}\}$, and L_s is the Glauber operator[2] that describes stochastic flips $s_x \to -s_x$. Thus, L_s generates from $\mathbf{s}$ a new configuration (denoted $\mathbf{s}^x$), and $c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\zeta})$ is the associated transition probability per unit time (*rate*) when the set $\boldsymbol{\zeta}$ is given. L_ζ induces stochastic changes of $\boldsymbol{\zeta}$ with rate $w(\boldsymbol{\zeta} \to \boldsymbol{\eta} \mid \mathbf{s})$ when the spin configuration is $\mathbf{s}$. Furthermore, (1.2) presumes that the processes governed by L_s and L_ζ have *a priori* frequencies Γ_s and Γ_ζ, respectively, and we have defined $\Gamma = \Gamma_\zeta / \Gamma_s$.

For simplicity, the elementary rates $c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\zeta})$ in (1.3) satisfy detailed balance, i.e.,

$$c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\zeta}) = c(\mathbf{s}^x \to \mathbf{s} \mid \boldsymbol{\zeta}) \exp[-\beta\, \Delta H_\zeta], \qquad \Delta H_\zeta \equiv H_\zeta(\mathbf{s}^x) - H_\zeta(\mathbf{s}) \tag{1.5}$$

where $\beta = (k_B T)^{-1}$, with respect to a class of Hamiltonians $H_\zeta(\mathbf{s})$ that we shall take to be of the NN Ising type (1.1). A simple choice is

$$c(\mathbf{s} \to \mathbf{s}^x \mid \zeta) = \phi(\beta\, \Delta H_\zeta) \tag{1.6}$$

where $\phi(X)$ is a positive function which is arbitrary except that $\phi(0) = 1$, and $\phi(X) \to 0$ as $X \to \infty$, and $\phi(X) = e^{-X}\phi(-X)$ to have (1.5). $\phi(X) = 1 - \tanh(\frac{1}{2}X)$, $\min(1, e^{-X})$, and $e^{-X/2}$ have been used to implement several stochastic Ising systems (see, e.g., ref. 20, and references therein). Our system (1.1)–(1.5) reduces to the familiar Glauber–Ising system[2] with (Gibbs) equilibrium states as long as $\Gamma \equiv 0$ and

$$\zeta = \{J_{xy} = J = \text{const},\ h_x = h = \text{const}\}$$

For $\Gamma \neq 0$ and $\zeta = \{J_{xy}, h_x\}$ differing from the latter choice, the system allows one to consider several interesting situations that are nonequilibrium in general.

We define two particular cases:

(i) The time evolution of ζ is independent of the time evolution of $\mathbf{s}$, i.e.,

$$w(\zeta \to \boldsymbol{\eta} \mid \mathbf{s}) = w(\zeta \to \boldsymbol{\eta}) \qquad \text{for any } \mathbf{s} \tag{1.7}$$

as if the changes of ζ were caused by some external, independent agent.

(ii) Some dependence between the processes (1.3) and (1.4) exists.

The latter may describe, for example, a system whose impurities (i.e., ζ) are canonically driven by a heat bath at inverse temperature, say β', that differs from the one for spin changes, β. In a sense, this generalizes the case of the annealed system; the explicit analysis of such a specific situation is beyond the scope of the present paper, and it will be reported elsewhere.

The present paper deals with case (i), i.e., we assume that (1.7) holds hereafter. It allows us to describe in Section 2 a changeover from the MQD to a class of *models with kinetic disorder* (MKD) that have been studied before.[18–22] More precisely, we find that $\Gamma \to 0$ corresponds to the (equilibrium) MQD, while $\Gamma \to \infty$ leads to the (nonequilibrium) MKD; two time scales that characterize the evolution with time of the impurity and spin configurations, respectively, are well differentiated from each other within the latter limit. Therefore, (1.1)–(1.7) help to elucidate the physical significance of the nonequilibrium situations studied before, and further questions such as the role of diffusion of impurities in deciding thermodynamics. Furthermore, we find novel interesting cases that are sometimes amenable to simple analytical treatment. On the other hand, certain situations included in our models may be interpreted as a class of neural networks as reported before.[23] To illustrate the formalism, we work out in Section 3 the explicit relation between two one-dimensional systems: a quenched spin-glass model by Mattis,[24] on the one hand, and a version of MKD that may be interpreted as a nonequilibrium spin-glass model,[18] i.e., a system in which a *dynamical* conflict between positive and negative couplings exists, on the other. The analysis of such a relation clarifies also, in particular, some aspects of time relaxation in the one-dimensional quenched spin-glass Ising model. It is remarkable that the MKD may exhibit an exceptionally rich class of nonequilibrium phase transitions and critical phenomena. That is, for example, various versions of the model (1.1)–(1.7) undergo within the limit $\Gamma \to \infty$ first- and/or second-order phase transitions that have no equilibrium counterpart, and the whole resulting picture is strongly influenced by kinetics, e.g., the latter may induce nonuniversal critical behavior. We describe some results for such a limit in Section 4; the focus there is on the classification of the elementary rates that we believe is a main problem in this field at present. We thus find a special class of rates for which the MKD can be solved exactly for any lattice dimension d. Finally, as a further illustration of the formalism in Section 2, we consider a microscopic realization of the concept of a *heat bath* in Section 5. That is, a version of our model system is explicitly shown to have the properties one should require for a *heat bath* system.

2. AUTONOMOUS DISORDER

The solution of the master equation (1.2) may be written as

$$P_t(\mathbf{s}, \zeta) = \Pi_t(\mathbf{s} \mid \zeta)\, p_t(\zeta) \tag{2.1}$$

where $\sum_s \sum_\zeta P_t(\mathbf{s}, \zeta) = 1$ and $\sum_s \Pi_t(\mathbf{s} \mid \zeta) = 1$. Assuming (1.7), it follows by substitution that

$$\partial p_t(\zeta)/\partial t = \Gamma L_\zeta\, p_t(\zeta) \tag{2.2}$$

$$\partial \Pi_t(\mathbf{s} \mid \zeta)/\partial t = L_s \Pi_t(\mathbf{s} \mid \zeta) + \Gamma p_t(\zeta)^{-1} \left[L_\zeta \Pi_t(\mathbf{s} \mid \zeta) - \Pi_t(s \mid \zeta)\, L_\zeta\right] p_t(\zeta) \tag{2.3}$$

Equation (2.2) says that the distribution of disorder at each time, $p_t(\zeta)$, depends only on the initial condition $p_0(\zeta)$ and on the intrinsic mechanism L_ζ when (1.7) holds; (2.3) indicates that $\mathbf{s}$ evolves coupled to $p_t(\zeta)$, in general. The coupling strength depends on the value of the parameter Γ that appears in (2.2) and (2.3). The study of several degrees of coupling follows.

Consider first the simple limiting case $\Gamma=0$; then, (2.2) implies that

$$p_t(\zeta)=p_0(\zeta) \qquad \text{for any} \quad t \tag{2.4}$$

and the equation that governs the conditional probability simply reduces to

$$\partial \Pi_t(\mathbf{s}\,|\,\zeta)/\partial t = L_s \Pi_t(\mathbf{s}\,|\,\zeta) \tag{2.5}$$

That is, the initial distribution of disorder, $p_0(\zeta)$, remains quenched or frozen-in with time for $\Gamma=0$, and the time evolution of $\mathbf{s}$ is then governed by (2.5), which is the familiar Glauber equation.[2] This situation is the one that characterizes the familiar MQD. That is, the standard bond-dilute, random-field, and spin-glass Ising models may be interpreted as realizations of our model (1.1)–(1.7) when both $\Gamma=0$ and the last function in (2.1) is independent of t as follows:

$$p_t(\zeta)=\prod_{|x-y|=1}[q\delta(J_{xy})+(1-q)\,\delta(J_{xy}-J_0)]$$

$$p_t(\zeta)=\prod_{x}\{q\delta(h_x)+\tfrac{1}{2}(1-q)[\delta(h_x-h_0)+\delta(h_x+h_0)]\}$$

$$p_t(\zeta)=\prod_{|x-y|=1}[\tfrac{1}{2}\delta(J_{xy}-J_0)+\tfrac{1}{2}\delta(J_{xy}+J_0)]$$

respectively. This finding is discussed in more detail in Sections 3 and 4.

Next, we define a new time scale, $\tau\equiv\Gamma t$, that remains finite within the limit $\Gamma\to 0$, $t\to\infty$. Then, Eq. (2.2) transforms into

$$\partial\tilde{p}_\tau(\zeta)/\partial\tau = L_\zeta\,\tilde{p}_\tau(\zeta), \qquad \tilde{p}_\tau(\zeta)\equiv p_{\Gamma^{-1}\tau}(\zeta) \tag{2.6}$$

On the other hand, one expects

$$L_s\tilde{\Pi}_\tau(\mathbf{s}\,|\,\zeta)\to 0 \quad \text{as} \quad \Gamma\to 0, \qquad \tilde{\Pi}_\tau(\mathbf{s}\,|\,\zeta)\equiv\Pi_{\Gamma^{-1}\tau}(\mathbf{s}\,|\,\zeta) \tag{2.7}$$

on the time scale τ because of (2.3). Equation (2.7) implies that

$$\tilde{\Pi}_\tau(\mathbf{s}\,|\,\zeta)=\Pi_{st}(\mathbf{s}\,|\,\zeta) \qquad \text{for any} \quad \tau>0 \tag{2.8}$$

which corresponds to the solution of (2.5). Thus, the general solution of (1.2) as $\Gamma\to 0$ is

$$\tilde{P}_\tau(\mathbf{s},\zeta)\equiv P_{\Gamma^{-1}\tau}(\mathbf{s},\zeta)=\Pi_{st}(\mathbf{s}\,|\,\zeta)\,\tilde{p}_\tau(\zeta) \tag{2.9}$$

where the last function is the solution of (2.6). We remark that the validity of (2.7) and, more generally, of the simplified description (2.9) rests upon the existence for $\Gamma\to 0$ of well-defined time scales that are characterized by time variables t and τ, respectively. More precisely, (2.7) requires that $\max_\zeta t_s(\zeta)\ll\Gamma^{-1}$, where $t_s(\zeta)$ is the relaxation time associated to (2.5).

A similar discussion applies to the opposite case, i.e., to the limit $\Gamma\to\infty$, $t\to 0$, namely, the solution of (1.2) may be written also as

$$P_t(\mathbf{s},\zeta)=\Pi'_t(\zeta\,|\,\mathbf{s})\,\mu_t(\mathbf{s}) \tag{2.10}$$

where $\sum_s \Pi'_t(\mathbf{s}\,|\,\zeta)=1$, and one gets by substitution if condition (1.7) holds that

$$\partial\mu_t(\mathbf{s})/\partial t=\sum_\zeta L_s\Pi'_t(\zeta\,|\,\mathbf{s})\,\mu_t(\mathbf{s}) \tag{2.11}$$

and

$$\partial\Pi'_t(\zeta\,|\,\mathbf{s})/\partial t=\Gamma L_\zeta\Pi'_t(\zeta\,|\,\mathbf{s})+\mu_t(\mathbf{s})^{-1}\,[L_s\Pi'_t(\zeta\,|\,s)-\Pi'_t(\zeta\,|\,\mathbf{s})\sum_\zeta L_s]\,\mu_t(s) \tag{2.12}$$

If we introduce the (finite) time scale $\tau\equiv\Gamma t$, the latter two equations may be written (using notation similar to the one before)

$$\partial\tilde{\mu}_\tau(\mathbf{s})/\partial\tau=\Gamma^{-1}\sum_\zeta L_s\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})\,\tilde{\mu}_\tau(\mathbf{s}) \tag{2.13}$$

$$\begin{aligned}\partial\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})/\partial\tau=L_\zeta\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})+\Gamma^{-1}\tilde{\mu}_\tau(\mathbf{s})^{-1}\,[L_s\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})\\ -\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})\sum_\zeta L_s]\,\tilde{\mu}_\tau(\mathbf{s})\end{aligned} \tag{2.14}$$

respectively. Thus,

$$\partial\tilde{\mu}_\tau(\mathbf{s})/\partial\tau=0, \qquad \partial\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})/\partial\tau=L_\zeta\tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s}) \tag{2.15}$$

within the limit $\Gamma\to\infty$, $t\to 0$. The latter two equations imply, respectively,

$$\tilde{\mu}_\tau(\mathbf{s})=\tilde{\mu}_0(\mathbf{s}) \quad \text{for any} \quad \tau, \qquad \tilde{\Pi}'_\tau(\zeta\,|\,\mathbf{s})=\tilde{p}_\tau(\zeta) \tag{2.16}$$

The latter equality is a consequence of the fact that the operator L_ζ is independent of $\mathbf{s}$ [cf. Eqs. (1.4) and (1.7)] and of the assumed uniqueness of the solution of the master equation (2.6). Consequently, (2.10) transforms into

$$\tilde{P}_\tau(\mathbf{s},\zeta)=\tilde{p}_\tau(\zeta)\,\tilde{\mu}_0(\mathbf{s}) \tag{2.17}$$

when $\Gamma\to\infty$ (for small enough values of t).

Finally, we consider the case Γ, $\tau \to \infty$ for finite values of $t = \tau\Gamma^{-1}$. We get

$$\Pi_t'(\zeta \mid \mathbf{s}) = p_{st}(\zeta), \qquad \partial\mu_t(s)/\partial t = \sum_{\zeta} p_{st}(\zeta)\, L_s \mu_t(s) \tag{2.18}$$

Namely, during the interval elapsed between two consecutive spin-flip processes, the other degrees of freedom will, in general, undergo enough changes when $\Gamma, \tau \to \infty$ to assure that $p_t(\zeta)$ has reached the steady value $p_{st}(\zeta)$. One needs to assume here also that L_ζ is such that $p_{st}(\zeta)$ is unique and may be reached from almost any initial condition on an interval that is small enough compared to the characteristic time associated to the evolution of the spin configuration.

Summing up, the system (1.1)–(1.6) has two interesting cases as long as (1.7) holds: (a) For $\Gamma \to 0$, spins remain frozen-in while impurities evolve according (2.6) *within the scale* $\tau = \Gamma t$ for large enough values of t. Within the original time scale t the limiting case $\Gamma = 0$ corresponds to the familiar MQD characterized by (2.4) and (2.5). (b) For $\Gamma \to \infty$, one may distinguish, in general, two well-separated time scales in the system evolution. On the one hand, a microscopic time scale τ exists in which the spins remain essentially frozen-in, while impurities evolve according to (2.15). On the other hand, a macroscopic time scale t exists in which impurities have already reached their stationary state, $p_{st}(\zeta)$. According to (2.18), the spin system evolves within the latter time scale as implied by an *effective Glauber operator*, namely

$$L_s^{\text{eff}} \equiv \sum_{\zeta} p_{st}(\zeta)\, L_s \tag{2.19}$$

This is the Glauber propagator L_s averaged with respect to the stationary solution of (2.12) as $\Gamma \to \infty$. Sections 4 and 5 contain several realizations of case (b); Section 3 is devoted to a model with varying Γ in which a changeover between cases (a) and (b) is observed explicitly.

3. THE MATTIS SPIN-GLASS MODEL

Mattis[24] introduced an equilibrium disordered system characterized by the Hamiltonian

$$H_\zeta(\mathbf{s}) = -\sum_x J_x s_x s_{x+1}, \qquad J_x = J_0 \zeta_x \zeta_{x+1} \tag{3.1}$$

for $d = 1$; here, J_0 is a constant and $\zeta_x = \pm 1$, $s_x = \pm 1$. The disorder is represented here by the random variables ζ_x, distributed spatially between sites according to

$$p(\zeta_x) = \tfrac{1}{2}\delta(\zeta_x - 1) + \tfrac{1}{2}\delta(\zeta_x + 1)$$

The one-dimensional case (3.1) corresponds to the Edwards–Anderson (quenched) spin-glass model[10] (but no correspondence occurs for $d > 1$). Thus, this is an interesting case that may be generalized to obtain a convenient explicit (one-dimensional) realization of our model. With this aim, we adopt the description (1.2)–(1.7) and define transition rates for the time evolution of the spin and disorder variables, respectively, as follows:

$$c(\mathbf{s} \to \mathbf{s}^x \mid \zeta) = 1 - \tanh[\beta J_0 s_x \zeta_x (s_{x+1}\zeta_{x+1} + s_{x-1}\zeta_{x-1})] \tag{3.2}$$

and

$$w(\zeta \to \zeta^x \mid \mathbf{s}) = \text{const} \qquad \text{for any} \quad \zeta, \mathbf{s} \tag{3.3}$$

The latter is the simplest representation of assumption (1.7). With (1.4), (3.3) states that the set ζ changes by *flipping*, i.e., $\zeta_x \to -\zeta_x$, completely at random, say, as driven by a heat bath at *infinite temperature*. On the other hand, we remark that (3.2) reduces the system to a simple case. In fact, the resulting state $(\mathbf{s}, \zeta)$ will be shown to be Bernoulli within the limit $\Gamma \to \infty$ for (3.2), and the system does not exhibit a zero-T critical point. The latter two (easy) features are not present in most of the one-dimensional MKD, e.g., each of the MKD in Section 4 corresponds to a more involved expression for $c(\mathbf{s} \to \mathbf{s}^x \mid \zeta)$. Our choice (3.2) is motivated by the fact that it makes the (generalized) kinetic system easily solvable while it still allows for some interesting behavior. For example, it makes the sets $\mathbf{s}$ and ζ correlated to each other (though no correlations exist within each of them) for any finite Γ ($\neq 0$); one recovers the original Mattis model (where correlations exist within both $\mathbf{s}$ and ζ) for $\Gamma = 0$.

To prove the latter and further facts, let us introduce the variables $\sigma_x \equiv \zeta_x s_x$. Then, one may write after Eqs. (1.2)–(1.4) and (3.3) that

$$\begin{aligned}\partial P_t(\boldsymbol{\sigma}, \zeta)/\partial t = \sum_{\sigma^x, \zeta^x} \{&\tilde{c}(\boldsymbol{\sigma}^x \to \boldsymbol{\sigma})\, P_t(\boldsymbol{\sigma}^x, \zeta) + \Gamma P_t(\boldsymbol{\sigma}^x, \zeta^x) \\ &- [\tilde{c}(\boldsymbol{\sigma} \to \boldsymbol{\sigma}^x) + \Gamma]\, P_t(\boldsymbol{\sigma}, \zeta)\}\end{aligned} \tag{3.4}$$

Here,

$$\tilde{c}(\boldsymbol{\sigma} \to \boldsymbol{\sigma}^x) = 1 - \tfrac{1}{2}\gamma\sigma_x(\sigma_{x+1} + \sigma_{x-1}) \tag{3.5}$$

where $\gamma \equiv \tanh(2\beta J_0)$, because of (3.2). One may write from (3.4) that

$$\begin{aligned}\partial\langle g\rangle_t/\partial t = \sum_{\sigma^x} \{&\langle [g(\boldsymbol{\sigma}^x, \zeta) - g(\boldsymbol{\sigma}, \zeta)]\, \tilde{c}(\boldsymbol{\sigma} \to \boldsymbol{\sigma}^x)\rangle_t \\ &+ \Gamma\langle g(\boldsymbol{\sigma}^x, \zeta^x) - g(\boldsymbol{\sigma}, \zeta)\rangle_t\}\end{aligned} \tag{3.6}$$

where

$$\langle g(\boldsymbol{\sigma}, \boldsymbol{\zeta})\rangle_t \equiv \sum_{\sigma} \sum_{\zeta} g(\boldsymbol{\sigma}, \boldsymbol{\zeta})\, P_t(\boldsymbol{\sigma}, \boldsymbol{\zeta}) \tag{3.7}$$

Then, it follows from (3.6) for $g(\boldsymbol{\sigma}, \boldsymbol{\zeta}) = \zeta_x$ that

$$\langle \zeta_x \rangle_t = \langle \zeta_x \rangle_0 \exp(-2\Gamma t) \tag{3.8}$$

which indicates an exponential relaxation to a stationary distribution of disorder with $\langle \zeta_x \rangle_{st} = 0$ for $\Gamma > 0$. Some information concerning the time evolution of the spin configuration follows from (3.6) for $g(\boldsymbol{\sigma}, \boldsymbol{\zeta}) = \zeta_0 \sigma_x$; one obtains

$$\partial F_t(x)/\partial t = 4\Gamma[\delta_{x,0} F_t(0) - F_t(x)] - 2F_t(x) + \gamma[F_t(x+1) + F_t(x-1)] \tag{3.9}$$

where $F_t(x) \equiv \langle \zeta_0 \sigma_x \rangle_t$ and $\delta_{x,y}$ is the Kronecker delta. After Fourier transforming, we obtain

$$\begin{aligned} F_t(0) = \langle s_0 \rangle_t = {} & e^{-2(1+2\Gamma)t} \sum_x F_0(x)\, I_{|x|}(2\gamma t) \\ & + 4\Gamma \int_0^t d\tau\, F_{t-\tau}(0)\, e^{-2(1+2\Gamma)\tau} I_0(2\gamma\tau) \end{aligned} \tag{3.10}$$

in particular, where $I_n(X)$ stands for the Bessel function.

Next, we remark that the differential equation (3.9) and the (Volterra) integral equation (3.10) reduce to the solution of the original Mattis model with quenched disorder as $\Gamma \to 0$. For instance, one obtains

$$\langle s_0 \rangle_t = e^{-2t} \sum_x \langle s_x \rangle_0 \zeta_0 \zeta_x I_{|x|}(2\gamma t) \tag{3.11}$$

if the initial condition $F_0(x) = \zeta_0 \zeta_x \langle s_x \rangle_0$ is used (which corresponds to having all the bonds fixed at $t = 0$). To deal with more general conditions, one may note that a solution $\langle s_0 \rangle_t$ of Eq. (3.10) always exists which is unique and may be represented by a convergent series (e.g., ref. 25); such a solution may be obtained from the Laplace transform of (3.10),

$$f(z) \equiv \int_0^\infty dt\, e^{-tz} F_t(0) = (\theta - 4\Gamma)^{-1} \sum_x F_0(x)[2\gamma/(\alpha + \theta)]^{|x|} \tag{3.12}$$

where $\alpha \equiv z + 2 + 4\Gamma$ and $\theta \equiv (\alpha^2 - 4\gamma^2)^{1/2}$.

Information concerning several interesting cases follows from (3.12). Consider first the infinite-temperature limit, which corresponds to $\gamma \to 0$. One gets from (3.12) that

$$f(z) = (z + 2)^{-1} \sum_x F_0(x) \tag{3.13}$$

which is independent of Γ. It implies

$$\langle s_0 \rangle_t = \langle s_0 \rangle_0 \exp(-2t) \tag{3.14}$$

i.e., the spin system relaxes at infinite temperature, independent of the other degrees of freedom, with a characteristic time given by $\tau_s = 1/2$, while the relaxation time for the disorder is $\tau_b = (2\Gamma)^{-1}$, according to (3.8). For small enough values of γ, one may develop (3.12) to write, after inverse Laplace transforming, that

$$\begin{aligned} \langle s_0 \rangle_t e^{2t} = {} & F_0(0) + \gamma(4\Gamma)^{-1}(1 - \varepsilon)[F_0(1) + F_0(-1)] \\ & - \gamma^2 (4\Gamma)^{-2} [\varepsilon(1 + 4\Gamma t) - 1][F_0(2) + F_0(-2)] \\ & + 2\gamma^2 (4\Gamma)^{-2} (4\Gamma t - 1 + \varepsilon) F_0(0) + O(\gamma^3) \end{aligned} \tag{3.15}$$

where $\varepsilon \equiv \exp(-4\Gamma t)$. The exponential e^{-2t} that multiplies the RHS here is effectively modulated by powers of t if $t < \frac{1}{2}\tau_b = (4\Gamma)^{-1}$, and the relaxation becomes then independent of Γ; i.e.,

$$\begin{aligned} \langle s_0 \rangle_t \approx e^{-2t} \{ & F_0(0) + \gamma t[F_0(1) + F_0(-1)] \\ & + \tfrac{1}{2}\gamma^2 t^2 [F_0(2) + F_0(-2) + 2F_0(0)] + O(\gamma^3) \} \end{aligned} \tag{3.16}$$

The latter behavior, which extends to any value of the time as $\Gamma \to 0$, corresponds to the peculiar relaxation that is known to characterize the standard spin-glass model. Only for $t > \frac{1}{2}\tau_b$, where $\tau_b \to 0$ as $\Gamma \to \infty$, is the system relaxation influenced, and even dominated, by the evolution of the bond distribution. Also noticeable is the fact that the system relaxation is determined anyhow by functions F_0 that describe initial correlations between the bond and spin configurations.

The case $\Gamma \to \infty$ may be studied by developing (3.12) for small Γ^{-1}; one obtains

$$\langle s_0 \rangle_t e^{2t} = \langle s_0 \rangle_0 (1 + 2\gamma^2 t \Gamma^{-1}) + \gamma(4\Gamma)^{-1} [F_0(1) + F_0(-1)] + O(\Gamma^{-2}) \tag{3.17}$$

That is, the bonds evolve as driven by a heat bath at infinite temperature, which is already a consequence of (3.3). The nonequilibrium spin-glass model[18] (cf. Section 4) is only recovered from the above generalized Mattis model when one considers the limit $\Gamma \to \infty$ besides condition (3.3), however. This specific finding illustrates the main general result in Section 2. That is, the MKD studied in refs. 18–22 involve (implicitly) essentially different time scales for the time changes in **s** and ζ in addition to a lack of correlations between impurities.

It appears worthwhile to study also the nature of the stationary correlations in the generalized model (1.2)–(1.4) with (3.2) and (3.3). Let us consider

$$\langle \sigma_{x_1}\sigma_{x_2}\zeta_{y_1}\zeta_{y_2}\rangle_t \equiv G(x_1, x_2; y_1, y_2) \tag{3.18}$$

where the average is defined in (3.7). It follows from (3.4) after some algebra that

$$\begin{aligned} &4\{\delta_{x_1,x_2} - \Gamma[2 - D(x_1, x_2, y_1, y_2)] - 1\}\, G(x_1, x_2; y_1, y_2) \\ &\quad + \gamma(1 - 2\delta_{x_1,x_2})[G(x_1, x_2+1; y_1, y_2) + G(x_1, x_2-1; y_1, y_2)] \\ &\quad + \gamma[G(x_1+1, x_2; y_1, y_2) + G(x_1-1, x_2; y_1, y_2)] = 0 \end{aligned} \tag{3.19}$$

Here,

$$\begin{aligned} D(x_1, x_2, y_1, y_2) \equiv{} & \delta_{x_1,x_2} + \delta_{x_1,y_2} + \delta_{x_2,y_2} + \delta_{y_1,y_2} + \delta_{x_1,y_1}(1 - 2\delta_{y_1,y_2}) \\ & \delta_{x_2,y_1}(1 - 2\delta_{x_2,y_2}) - 2\delta_{x_1,x_2}(\delta_{x_2,y_1} + \delta_{x_2,y_2} - 2\delta_{x_2,y_1}\delta_{x_2,y_2}) \end{aligned} \tag{3.20}$$

and the condition $\langle \sigma_x\sigma_x\zeta_y\zeta_y\rangle = 1$ has been used. For $\Gamma = 0$, (3.19) reduces to

$$G(x_1, x_2; y_1, y_2) = \zeta_{y_1}\zeta_{y_2}\eta^{|x_1 - x_2|} \tag{3.21}$$

where $\eta \equiv \tanh(\beta J)$, which is precisely the behavior of correlations in the Mattis case.

Otherwise, one may study the correlations numerically using (3.19). We have studied $G(x_1, x_2; y_1, y_2)$ by performing up to 10^3 iterations on a 20^4 hypercube with boundary conditions fixed to $G = 0$. The Mattis equilibrium correlations have been used as initial conditions. The accuracy of the method was tested by comparing the numerical estimate for $\Gamma = 0$ with the exact solution (3.21); the only systematic deviations occur for $|x_1 - x_2| > 9$ (moreover, one finds that the correlation length is always smaller than 6). We have concluded in this way that

$$G(x_1, x_2; x_1, x_2) = \langle s_{x_1}s_{x_2}\rangle = \delta_{x_1,x_2} \tag{3.22}$$

and

$$G(x, x; x_1, x_2) = \langle \zeta_{x_1}\zeta_{x_2}\rangle = \delta_{x_1,x_2} \tag{3.23}$$

according to our comment above that (3.2) makes the system exceptionally simple]. The correlations between the sets **s** and ζ, as measured by

$$G(n) \equiv G(x_1, x_2; y, y) = \langle s_{x_1}s_{x_2}\zeta_{x_1}\zeta_{x_2}\rangle, \qquad n \equiv |x_1 - x_2| \tag{3.24}$$

are nontrivial, however. This is illustrated by Figs. 1–3. The figures reveals the following facts:

1. $G(n)$ decreases from the Mattis value with increasing Γ for any given $n > 0$ (cf. Fig. 1; see also inset for Fig. 3). This agrees with our expectation that the sets **s** and ζ should become essentially uncorrelated (except at very short distances) for the nonequilibrium case, i.e., within the limit $\Gamma \to \infty$, for the simple choice (3.2).

2. $G(n)$ increases with decreasing temperature for any $n > 0$ (cf. Fig. 2), as one should expect to be the case in general.

3. $G(n)$ decays exponentially with n for any value of Γ and T (cf. insets for Figs. 1 and 2). In fact, $G(n) \approx 0$ for $n > 5$ independent of Γ (cf. Fig. 1), except at very low temperatures (cf. Fig. 2).

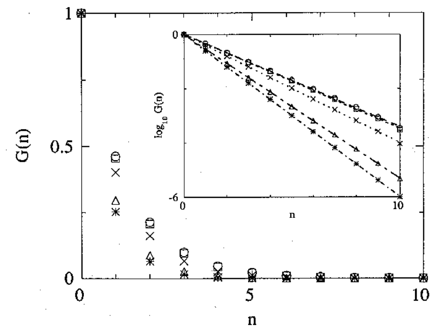

Fig. 1. The correlation function $G(n) \equiv G(x_1, x_2; y, y)$ (cf. Eq. (3.24)) for the generalized Mattis model in Section 3 as a function of $n \equiv |x_1 - x_2|$ for a given temperature, $T = 2$, and different values of Γ; namely, $\Gamma = 0$ (circles), 0.01 (squares), 0.1 (crosses), 0.4 (triangles), and 0.6 (asterisks). The inset is a semilogarithmic plot of the same data.

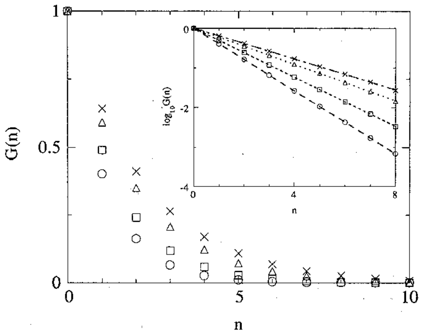

Fig. 2. The same as in Fig. 1, but for $\Gamma = 0.1$, and $T = 0.2$ (crosses), 1 (triangles), 1.5 (squares), and 2 (circles).

4. The correlation length ξ, as defined by

$$G(n) \sim \exp(-n/\xi) \qquad \text{for large } n \tag{3.25}$$

is practically constant for $T < 0.6$ (when $\Gamma = 0.1$), while it otherwise decreases with T (cf. Fig. 3). This is consistent with the fact that the case $\Gamma \neq 0$ does not have a zero-T critical point [for the choice (3.2)].

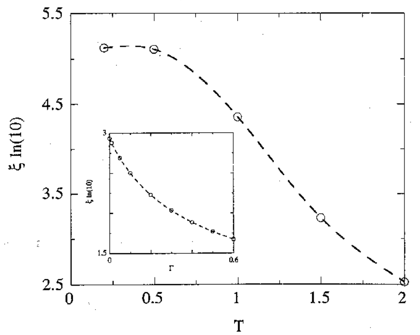

Fig. 3. The correlation length ξ, defined as the slope of the fits to the data in the inset for Figs. 1 and 2 (times ln 10), when $\Gamma = 0.1$ as a function of T (main graph), and as a function of Γ for $T = 2$ (inset).

4. THE LIMIT $\Gamma \to \infty$

This section deals with the model (1.1)–(1.7) within the limit $\Gamma \to \infty$. It follows from Section 2 that, on the appropriate time scale, this is equivalent to the MKD. That is, such a limit may be defined also as an Ising system in which the probability of **s** at time t, $\mu_t(\mathbf{s})$, satisfies

$$\partial \mu_t(\mathbf{s})/\partial t = \sum_{\mathbf{s}^x \in S} [c'(\mathbf{s}^x \to \mathbf{s})\, \mu_t(\mathbf{s}^x) - c'(\mathbf{s} \to \mathbf{s}^x)\, \mu_t(\mathbf{s})] \tag{4.1}$$

where the rate defines a competing process. Namely, **s** changes stochastically with probability $c'(\mathbf{s} \to \mathbf{s}^x)$ per unit time as due to the simultaneous action of independent spin-flip mechanisms:

$$c'(\mathbf{s} \to \mathbf{s}^x) = \langle\!\langle c(\mathbf{s} \to \mathbf{s}^x \mid \zeta) \rangle\!\rangle \equiv \int_{-\infty}^{+\infty} d\zeta\, p(\zeta)\, c(\mathbf{s} \to \mathbf{s}^x \mid \zeta) \tag{4.2}$$

The systems defined by Eqs. (4.1) and (4.2), which are to be complemented with (1.5), (1.6), and (1.1), correspond to MQD that have been studied in refs. 18–22. The case (4.1)–(4.2) reduces to the Glauber–Ising[(2)] model for $p(\zeta) = \delta(\zeta - \zeta_0)$, where ζ_0 is a constant and δ is the Dirac delta function. Otherwise, a conflict between different values of ζ (e.g., J's) occurs that leads asymptotically to a steady nonequilibrium state in general, as if a non-Hamiltonian agent is acting on the spin system. Consequently, the resulting steady state depends on details of kinetics, e.g., $H_\zeta(\mathbf{s})$, $p(\zeta)$, and $\phi(X)$ besides T.

Two different interpretations of (4.1)–(4.2) are possible:

1. The disorder variable (e.g., J or h) changes at each kinetic step so that it takes the same value chosen at random from distribution $p(\zeta)$ throughout the system. This suggests how to implement the MKD sometimes in the laboratory. For example, a magnetic system under a very rapidly fluctuating magnetic field may correspond to the MKD with $p(\zeta) \equiv p(h)$. With this aim one needs to vary randomly the applied field according to $p(h)$. Alternatively, one may produce regular variations of the field but with a period that is shorter than the mean time between successive transitions that modify the spin configuration.[(21)]

2. The fact that the involved spin-flip process has a local nature and the restriction to NN interactions in (1.1) allow for a different interpretation of (4.1)–(4.2). One may assume that only the disorder variables that directly concern the spin(s) involved by each transition $\mathbf{s} \to \mathbf{s}^x$ (e.g., the 2D bonds J_{xy}, $|\mathbf{x} - \mathbf{y}| = 1$, ending at s_x, or the local field h_x) change at random successively during time evolution. As demonstrated in Section 2, this

produces asymptotically a spatial distribution of disorder, say $p'(\zeta)$, which has two interesting properties:

(i) Within the appropriate time scale, $p'(\zeta)$ is similar *at each time* to the (quenched) random spatial distribution in the MQD.

(ii) Unlike the latter case, however, the disorder variable at each spatial location is not constant with time, but continuously changes at random according to the distribution $p(\zeta)$ in (4.2). This variation represents very fast and random diffusion of disorder that one may consider in turn as a simple representation of the sort of diffusion of disorder that occurs in natural substances due to atomic migration.

One may prove thermodynamics is the same for these two interpretations except *energy* fluctuations that are anomalously large for interpretation 1.[21] It reflects that the two cases correspond to different physical situations, but differences are more conceptual than practical, at least for (1.1). In particular, both cases have the same Monte Carlo implementation.[26]

The systematic study of the MKD[18–22] by several methods has revealed a rich class of steady states, phase transitions, and critical phenomena, as mentioned above. We discuss some known results for several specific realizations of the MKD in the remainder of this section. Our goal is not only completeness, i.e., to report on the interesting behavior of the general system in Section 1 within the limit $\Gamma \to \infty$, but we attempt a general classification of elementary transition rates which we believe is an important issue here at the present moment. Moreover, the important result then follows that, for a given class of rates, the MKD have a simple representation that may be solved exactly in some cases for any d-dimensional lattice, even for more general choices of *Hamiltonians* than (1.1). This is a rare case; it stresses the essential role played by the rate in determining the most important features of the steady state.

The simplest case of MKD occurs when the *effective rate* (4.2) satisfies *global detailed balance*. Namely, for

$$c'(\mathbf{s} \to \mathbf{s}^x) \exp[-E(s)] = c'(\mathbf{s}^x \to \mathbf{s}) \exp[-E(\mathbf{s}^x)] \tag{4.3}$$

where $E(\mathbf{s})$ is defined such that

$$\mu_{st}(\mathbf{s}) = \exp[-E(\mathbf{s})] \left\{ \sum_{s \in S} \exp[-E(\mathbf{s})] \right\}^{-1} \tag{4.4}$$

where $\mu_{st}(\mathbf{s})$ represents the stationary solution of (4.1). Since (4.3) holds, which restricts in practice the choices for $p(\zeta)$ and $\phi(X)$, the following result[27,28] applies:

The steady states for any one-dimensional MKD which is defined by Eqs. (4.1), (4.2), (1.5), and (1.6), where any of the functions $H_\zeta(\mathbf{s})$ that are involved by the latter has the NN Ising structure (1.1), may be characterized by an *effective Hamiltonian* $E(\mathbf{s})$ that has the short-range structure of (1.1).

More explicitly, one has (*only*) under these conditions that

$$E(\mathbf{s}) = -K_e \sum_{|x-y|=1} s_x s_y - \beta h_e \sum_x s_x \tag{4.5}$$

where

$$K_e = \frac{1}{4} \ln \left[\frac{\langle\!\langle \phi(-4K-2h) \rangle\!\rangle \langle\!\langle \phi(-2h) \rangle\!\rangle}{\langle\!\langle \phi(4K+2h) \rangle\!\rangle \langle\!\langle \phi(-2h) \rangle\!\rangle} \right] \tag{4.6}$$

with $K \equiv \beta J$, and

$$\beta h_e = \tfrac{1}{2} \ln[\langle\!\langle \phi(-2h) \rangle\!\rangle / \langle\!\langle \phi(2h) \rangle\!\rangle] \tag{4.7}$$

The symmetry (4.3) does not hold in one dimension for most choices of $p(\zeta)$ and $\phi(X)$.[28] It seems reasonable to expect that the system has a more interesting, say *full* nonequilibrium behavior in the absence of such a simplifying symmetry.[2] Thus, it is remarkable that the behavior of a system which has a *canonical* structure (4.5) is relatively complex, as suggested below. Formally, this is a consequence of the fact that the *effective parameters* (4.6) and (4.7) that describe the effect of conflicting kinetics involve in practice a complex dependence on (e.g.) temperature. A description of the most interesting behavior found for this *quasicanonical* class follows. Eventually, we refer also to some cases that lack property (4.5) as described by kinetic mean-field theory[19,22] and Monte Carlo simulations.[26,29]

An interesting case of MKD is the *nonequilibrium spin-glass system*.[18] It occurs (e.g.) if $H_\zeta(\mathbf{s})$ has the structure in (1.1) with $h_x = h = 0$, and $J_{xy} = J$ (which corresponds to ζ) is a random variable whose distribution $p(J) \equiv p(\zeta)$ allows for a competition between ferromagnetic and antiferromagnetic bonds. It is convenient, and apparently of some physical relevance, to distinguish two classes of rates, i.e., of functions ϕ in (1.6) as follows:

[2] It may occur that such a *full* nonequilibrium case can be characterized by a short-ranged effective Hamiltonian as well. In that case, $E(s)$ would only satisfy a condition that is weaker than (4.3). One may attempt to find an expression for $E(s)$ in this case by making a perturbative expansion either around the equilibrium case or around the state with symmetry (4.3), but this is beyond the scope of the present paper.

Hard dynamics. The rate is such that $\phi(-X)$ goes to a positive constant as $X \to \infty$, as for the familiar cases $\phi(X) = 1 - \tanh(\frac{1}{2}X)$ and $\phi(X) = \min(1, e^{-X})$.

Soft dynamics. The rate has the factorization property $\phi(\{\lambda_\alpha\}) = \prod_\alpha \phi_\alpha(\lambda_\alpha)$, where each factor satisfies the detailed balance (1.5), i.e., $\phi_\alpha(\lambda_\alpha) = \exp(-\lambda_\alpha)\,\phi_\alpha(\lambda_\alpha)$. As long as one has (1.1) in (1.6), so that the argument of ϕ is $\sum_\alpha \lambda_\alpha s_\alpha$, it follows that

$$\phi\left(\sum_\alpha \lambda_\alpha s_\alpha\right) = \left[\prod_\alpha \phi_\alpha(\lambda_\alpha)\exp(\tfrac{1}{2}\lambda_\alpha)\right]\exp\left(-\tfrac{1}{2}\sum_\alpha \lambda_\alpha s_\alpha\right) \qquad (4.8)$$

Due to factorization, the probability of flipping any spin is the superposition of independent processes, one for each bond ending at the spin. That is, some correlations are suppressed within the dynamical process. Therefore, the resulting behavior is particularly simple in some cases (but the present problem is an exception), e.g., (4.5) may hold even for $d > 1$, which justifies the name given to this class. The familiar rates $\phi(X) = e^{-X/2}$ and $\phi(X) = e^{-X/2}[\cosh(K)]^{-2d}$ belong to this class. One also has the property that $\phi(-X) \to e^{\pi X}$, where $\pi < 1$, for $X \to \infty$.

Consider the one-dimensional case for

$$p(J) = (1+\mu)^{-1}\,[\mu\delta(J - J_1) + \delta(J - J_2)], \qquad J_1 > 0 \quad J_2 < 0 \qquad (4.9)$$

where $\mu \equiv |J_2|/J_1 \in [0, \infty]$, which is a relatively general distribution of zero mean. One obtains (4.5) with $h_e = 0$, and K_e depending on the rate.[18] Within the limit $T \to 0$, it follows for hard rates that $K_e \approx \frac{1}{4}\ln\mu$; it indicates that the zero-T critical point that characterizes the pure system is washed out for $\mu \neq 0$ by dynamical disorder which makes the system very *hot* even at low temperatures. On the contrary, for soft rates one gets that $K_e \approx \pi(1-\mu)\,J_1/k_B T + \frac{1}{4}\ln\mu$. Therefore, critical behavior exists as $T \to 0$ for soft rates which may be characterized by exponents $\nu = 2(1-\mu)\,\pi$ and $\alpha = 2(1-\mu)\,\pi$, for example. That is, exponents depend not only on π, which describes the rate asymptotically, but also on μ, which refers to the symmetry of the disorder distribution (alternatively, one may describe the system by *pure* exponents if one allows for a variable characterizing the distance to the critical point, which is system dependent). It is worthwhile to emphasize that $\mu = 1$, which makes the exponents vanish, corresponds to the highest symmetry of (4.9), while a continuum of *impure* values for the exponents occurs otherwise in one dimension. The case $d > 1$ has been studied in detail by a kinetic mean-field method[19] that suggests a very rich phase diagram. On the other hand, a preliminary Monte Carlo study[26] for hard rates, $p(J) = q\delta(J - J_0) + (1-q)\,\delta(J + J_0)$, and $d = 2$ and 3 reveals the existence of a ferromagnetic-like phase for $q \geqslant q_0$ ($q_0 \approx 0.928$ for $d = 2$ and $q_0 \approx 0.835$ for $d = 3$) and of an antiferromagnetic-like phase for $q \leqslant 1 - q_0$. The corresponding critical exponents cannot be distinguished in practice from the Ising ones, except very near q_0, where some systematic deviations are observed. A sort of order with zero magnetization has been detected also between the two phases at low temperature, e.g., for $q = 0.91$ and $T < 0.6$ when $d = 2$. Moreover, the system exhibits the onset of percolation-like phenomena at $T = 0$ for $q = q_0$ that seems characterized for $d = 2$ and 3 by Ising critical exponents.[29]

The *nonequilibrium dilute magnetic system* is a (slightly) varying case of the previous system when distribution $p(\zeta) \equiv p(J)$ in (4.2) allows for competition between 0 and $J_0 > 0$, $p(J) = q\delta(J - J_0) + (1-q)\,\delta(J)$.[20] Again, hard rates preclude the existence of a zero-T critical point, which is similar to the situation for equilibrium where the one-dimensional Ising model loses the critical point for any nonzero concentration of nonmagnetic impurities. One may argue as follows: For the indicated bond distribution, *disorder* ($J = 0$) and *order* ($J = J_0$) occur on time scales which are proportional to $(1-q)^{-1}$ and $[q\phi(\beta J_0)]^{-1}$, respectively. Thus, condition $q\phi(\beta J_0) \gg 1 - q$ guaranties stability of the critical point, which is not satisfied for hard rates where time scales are comparable. On the other hand, the system presents a percolation-like critical point for $T = 0$, as for the nonequilibrium spin-glass model above. The situation is also similar to the one in the nonequilibrium spin-glass model when soft rates are used instead: the familiar critical point remains under dynamical disorder [in fact, condition $q\phi(\beta J_0) \gg 1 - q$ holds], and both thermal and percolation exponents may be defined that depend on details of kinetics. Further interesting cases have been analyzed elsewhere.[20]

The *nonequilibrium random field system*[21] is defined by setting $J_{xy} = J$ and $h_x = h$ in (1.1), and interpreting J as a constant and $h \equiv \zeta$ as a random variable. The ensuing macroscopic behavior differs essentially from the one for the models above due to the mathematical peculiarities induced by the presence of a field here. In particular, the one-dimensional case exhibits a critical point if one of the following situations arises:

(a) One has $h_e = 0$ as a consequence of the specific realization for $p(h)$ and $\phi(X)$, and K_e diverges as $T \to 0^+$. This occurs when $p(h) = p(-h)$ for the soft rate $\phi(X) = e^{X/2}$, for example. The resulting critical behavior is the same as for the pure Ising model, i.e., a zero-T critical point exists, unlike for the one-dimensional *quenched* random field Ising model. Furthermore, a fluctuation-dissipation relation does not hold, due to the existence of some added fluctuations.

(b) Both $h_e \to \infty$ and $K_e \to \infty$ as $T \to 0^+$, which has no equilibrium counterpart. It has been shown to occur only for soft rates, which imply

$h_e \neq 0$ in general. In particular, the choice $\phi(X) = 1 - \tanh(\frac{1}{2}X)$ induces two kinds of novel interesting critical behavior for $p(h) = \frac{1}{2}\delta(h - [\mu + \kappa]) + \frac{1}{2}\delta(h - [\mu - \kappa])$: namely, one has $\beta\delta' = 1 - \kappa/J$ (while $\beta\delta' = 1$ for the pure system) as $\mu \to 0$, and a line of critical points, which are characterized by $\nu = \min[1, (\kappa - \mu)/J]$, as long as $\mu < \kappa$; here, δ' represents the critical isotherm exponent.

(c) K_e has a more complex dependence on T, and none of the above cases occur. For example, hard rates and $p(h) = \frac{1}{2}q\delta(h - \kappa) + \frac{1}{2}q\delta(h + \kappa]) + (1 - q)\,\delta(h)$ may lead to $K_e \to K^0$ as $T \to 0^+$, where K^0 is either a positive constant (the disorder avoids any critical behavior), or a negative constant (the field competition induces an effective antiferromagnetic situation), or it is infinite (corresponds to a zero-T critical point), depending on the model version.

The (kinetic) mean-field study of this is interesting.[22] For $d > 1$, a zeroth-order approximation reveals that the hard rate $\phi(X) = 1 - \tanh(\frac{1}{2}X)$ produces a *tricritical point* that separates second- from first-order phase transitions, while the transitions are always of first order for $\phi(X) = \min(1, e^{-X})$, which is also *hard*. The phase diagram in the former case is consistent with a mean-field computation of the partition function for the quenched system by Aharony[30] within the same order of approximation. The study of hard rates within a first-order mean-field description reveals the existence of a tricritical point for $d > 2$ only, while the phase transition is always of second order for $d \leqslant 2$; the study of the quenched random-field Ising model within the same order of approximation[31,32] reveals a behavior qualitatively similar, except for some details, e.g., the quenched system has a tricritical point even for $d = 2$.

Summing up, the dynamical conflict in MKD caused by the superposition between several canonical spin-flip mechanisms may induce interesting novel behavior that deserves further investigation, specially for $d > 1$. In addition to this $\Gamma \to \infty$ limit, it would be interesting to study cases with finite Γ. It is already clear, however, that the symmetries within the effective rate (4.2) (which involves both the elementary rate with a given *Hamiltonian* function and the distribution and nature of the disorder variable) are here as conclusive for macroscopic behavior, at least, as the spatial symmetries that are familiar from equilibrium theory. It was shown[33] that the ordinary Ising fixed point is locally stable with respect to small amounts of irreversibility for systems with short-ranged interactions which respect the symmetry of the lattice and exhibit symmetry under spin inversion. This is consistent with the occurrence of different universality classes for full nonequilibrium systems, however. It corresponds to the existence of other stable fixed points with a domain of attraction

different from that of the Ising model, which is not excluded by the perturbative renormalization group argument in ref. 33. In fact, it seems that such a possibility may also be worked out exactly for $d > 1$ for a soft rate such as (4.8).[34]

5. MICROSCOPIC STRUCTURE OF A HEAT BATH

Finally, we work out in this section a further realization of our general system in Section 1.

From a macroscopic point of view, a heat bath is characterized by the inverse temperature parameter β. Within the context of kinetic lattice models, a heat bath which is coupled to the spin configuration of energy $H_0(\mathbf{s})$ induces stochastic changes of $\mathbf{s}$ as implied, for instance, by the Glauber operator L_s in (1.3), which involves a transition rate $c(\mathbf{s} \to \mathbf{s}^x)$. In order to guarantee that the latter leads asymptotically from almost any initial condition to a Gibbs state, $\mu_{st}(\mathbf{s}) \propto \exp[-\beta H_0(\mathbf{s})]$, it is sufficient to require detailed balance, i.e.,

$$c(\mathbf{s} \to \mathbf{s}^x) = f(\mathbf{s}; \mathbf{x})\{1 - \tanh[\beta(H_0(\mathbf{s}^x) - H_0(\mathbf{s}))]\} \tag{5.1}$$

where $f(\mathbf{s}^x; \mathbf{x}) = f(\mathbf{s}; \mathbf{x})$; cf. (1.5). Further details concerning the heat bath are irrelevant. Consequently, the set of variables or *phonon system* that characterizes the bath microscopic state, to be denoted $\boldsymbol{\eta}$ or $\boldsymbol{\zeta}$ hereafter, may be assumed to evolve in an appropriate, microscopic time scale by some sort of stochastic process, e.g., as stated by (1.4). In principle, one has no criteria to specify the associated transition rate $w(\boldsymbol{\eta} \to \boldsymbol{\zeta} \mid \mathbf{s})$. A case of our model in Sections 1 and 2 may be devised, however, that corresponds to a proper realization of a heat bath. Such a possibility and some interesting consequences of it are analyzed in the present section.

Consider the system (1.2)–(1.4) with transition rates $c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\zeta})$ and $w(\boldsymbol{\zeta} \to \boldsymbol{\eta} \mid \mathbf{s})$ whose specific form is unknown *a priori*. It has been shown in Section 2 that two different time scales may be defined naturally within the limit $\Gamma \to \infty$. Hence, we may interpret that the microscopic state of the heat bath or phonon configuration evolves toward its stationary state on a *microscopic time scale*, while the spin configuration is driven on a *macroscopic time scale* by an effective stochastic process. The latter is expected to be characterized by a transition rate, say $c_{\text{eff}}(\mathbf{s} \to \mathbf{s}^x)$, which depends on the (microscopic) stationary state of the bath, as stated in Eq. (2.19). Consistent with this, let us assume (for the sake of simplicity only) that:

(a) Within the limit $\Gamma \to \infty$, one has $\mu_{st}(\mathbf{s}) \propto \exp[-\beta H_0(\mathbf{s})]$, and condition (5.1) holds for the effective rate $c_{\text{eff}}(\mathbf{s} \to \mathbf{s}^x)$.

(b) The time evolution of $\boldsymbol{\eta}$ occurs independently from that of the spin configuration on the microscopic time scale, i.e., (1.7) holds.

(c) Only the transitions $\mathbf{s} \to \mathbf{s}^x$ that decrease the global energy of the system, say $H(\mathbf{s}, \boldsymbol{\eta})$, are allowed for a given microscopic state $\boldsymbol{\eta}$ and, consequently, for a given energy of the bath.

Under conditions (a)–(c), the nature of the heat bath or, more precisely, the unknowns $c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\eta})$ and $w(\boldsymbol{\zeta} \to \boldsymbol{\eta} \mid \mathbf{s})$, may be specified. Conditions (b) and (c) are convenient in order to determine a specific family of transition rates out of infinitely many having the right limiting property (a).

The global Hamiltonian for the spin system coupled to the bath may be written as

$$H(\mathbf{s}, \boldsymbol{\eta}) = H_0(\mathbf{s}) + H_1(\mathbf{s}, \boldsymbol{\eta}) + H_2(\boldsymbol{\eta}) \tag{5.2}$$

where H_2 represents the energy of the phonons, and H_1 is the energy associated with the interaction between the spin and phonon systems. For simplicity, one may take

$$H_1(\mathbf{s}, \boldsymbol{\eta}) = -\tfrac{1}{2}\lambda^{-1}\eta \sum_x s_x \tag{5.3}$$

where the bath state is characterized for simplicity by a single variable η, and λ characterizes the coupling between the spin and phonon systems. Consistently with condition (c), we propose

$$c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\eta}) = f_x(\mathbf{s}, \boldsymbol{\eta})\{1 - \mathrm{sgn}[H(\mathbf{s}^x, \boldsymbol{\eta}) - H(\mathbf{s}, \boldsymbol{\eta})]\} \tag{5.4}$$

for the rate governing the evolution of the spin system given $\boldsymbol{\eta}$; here, $\mathrm{sgn}(X) = 1$ for $X \geqslant 0$ and $\mathrm{sgn}(X) = -1$ for $X < 0$. On the other hand, we assume that, if isolated, i.e., $H_1 = 0$, the steady-state distribution of the bath is $p_{st}(\boldsymbol{\eta}) \propto \exp\{-H_2(\boldsymbol{\eta})\}$, and the associated dynamics may be obtained from the detailed balance condition:

$$w(\boldsymbol{\zeta} \to \boldsymbol{\eta})\, p_{st}(\boldsymbol{\zeta}) = w(\boldsymbol{\eta} \to \boldsymbol{\zeta})\, p_{st}(\boldsymbol{\eta}) \tag{5.5}$$

Thus, one needs to determine both $H_2(\boldsymbol{\eta})$ and $f_x(\mathbf{s}, \boldsymbol{\eta})$ to characterize completely the two dynamical processes (5.5) and (5.4), respectively. Then the condition (a) that $c_{\mathrm{eff}}(\mathbf{s} \to \mathbf{s}^x)$ needs to satisfy (5.1) for $\Gamma \to \infty$ leads to a relation between H_2 and $f_x(\mathbf{s}, \boldsymbol{\eta})$: namely, one obtains that

$$f_x(\mathbf{s}, \boldsymbol{\eta}) = f(\mathbf{s}; \mathbf{x})[p_{st}(\boldsymbol{\eta})\, e^{\eta}(1 + e^{-\eta})^2]^{-1} \tag{5.6}$$

where

$$f(\mathbf{s}, \mathbf{x}) = \int_{-\infty}^{\infty} d\boldsymbol{\eta}\, p_{st}(\boldsymbol{\eta})\, f_x(\mathbf{s}, \boldsymbol{\eta}) \tag{5.7}$$

Therefore, the type of thermal bath, i.e., $p_{st}(\boldsymbol{\eta})$ or H_2, determines the microscopic mechanisms. In particular, if one chooses

$$p_{st}(\boldsymbol{\eta}) = \left(2 \cosh \frac{\eta}{2}\right)^{-2} \tag{5.8}$$

it follows that

$$c(\mathbf{s} \to \mathbf{s}^x \mid \boldsymbol{\eta}) = f(\mathbf{s}; \mathbf{x})(1 - \mathrm{sgn}\{\lambda[H_0(\mathbf{s}^x) - H_0(\mathbf{s})] + \eta s_x\}) \tag{5.9}$$

and

$$w(\eta \to \zeta) = \Psi[2 \ln(\cosh \tfrac{1}{2}\zeta / \cosh \tfrac{1}{2}\eta)] \tag{5.10}$$

where $\Psi(X) = \Psi(-X)\, e^{-X}$

Summing up, if the basic equations (1.2)–(1.4) are complemented with (5.9) and (5.10), the resulting model corresponds precisely to the usual concept of a bath as long as $\Gamma \to \infty$. Thus, finite values of Γ correspond in a sense to a generalization of that concept in which the two involved time scales may be comparable. Such a generalization may have some interesting applications, but this is not investigated here.

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