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MAGNETIC SYSTEM UNDER A FLUCTUATING FIELD*

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We study nonequilibrium steady states, phase transitions and critical phenomena in a d-dimensional lattice model which represents a magnetic system under the action of a field fluctuating very rapidly with time. This induces competing kinetics which produces a sort of (dynamical) frustration which might occur also in some natural disordered systems. The exact solution for $d = 1$, partial exact results for $d \geq 2$, and a comparison with some related models are reported.

KEY WORDS: Steady nonequilibrium states, nonequilibrium phase transitions, Ising-like magnetic systems, dilute antiferromagnets, master equation with competing kinetics.

The quenched random-field Ising model (QRFIM) (Imry and Ma, 1975; see also Imry, 1984, and Natterman and Villain, 1988, for reviews) is a ferromagnetic Ising system whose spins at different sites suffer random local magnetic fields h independently assigned according to some distribution $f(h)$, e.g., $f(h) = \frac{1}{2}\delta(h - h_0) + \frac{1}{2}\delta(h + h_0)$, where δ represents the Dirac delta function and h_0 is a constant. This is a paradigm of a disordered system questioning concepts and techniques in the theory of critical phenomena, which is rather concerned with pure systems occurring for $f(h) = \delta(h - h_0)$. The QRFIM has also interested experimentalists (see, for instance, Belanger, Rezende, King and Jacarino, 1985; Birgeneau, Shapira, Shirane, Cowley and Yoshizawa, 1986) after the recognition (Fishman and Aharoni, 1979) that it may describe true physical systems, namely, dilute antiferromagnets in a uniform field whose spins are present at each lattice site with a probability which is independent of other spins. In spite of this activity, exact results are rare and partial at present (see, for instance, Derrida, Vannimenus and Pomeau, 1978; Grinstein and Mukamel, 1983; Bruinsma and Aeppli, 1983; Bricmont and Kupiainen, 1988; Wher, 1989, for some related work). Consequently, there still remain some doubts concerning both the nature of an ordered phase which may possibly exist in the QRFIM for $d > 2$, and the relevance of the various models proposed to analyze a large amount of existing experimental data exhibiting *unusual* behaviour. In relation to the latter, some of the difficulties revealed by the standard (equilibrium) analysis of random-field

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systems seem to be associated to the fact that their macroscopic behaviour is influenced by kinetics and has other nonequilibrium features.

The study of other impure materials is hampered by similar difficulties. This has been the main motivation to model recently spin glasses (Garrido and Marro, 1991) and magnetically dilute systems (Garrido and Marro, 1992) by adopting a point of view which differs from the standard consideration of quenched disorder. That is, one may argue that impurities are not frozen-in in natural systems but keep moving in time due to atomic migration; unlike in the familiar annealed systems, however, such a conceivable diffusion of disorder in real matter probably occurs rather independently of the other (spin) degrees of freedom. Following this philosophy, we study here Ising-like systems whose kinetics involve a random competition between various external magnetic fields. This generates a sort of *dynamical frustration* whose consideration might be relevant to the understanding of some of the peculiarities exhibited by random-field systems in nature, given the situation described above. In any case, our system is also interesting because it represents a magnetic system under a randomly fluctuating magnetic field, e.g., one which varies regularly with a much shorter period than the mean time between successive transitions modifying the spin configuration. Moreover, the competition which characterizes our model system is in a sense equivalent to a non-Hamiltonian constraint which prevents the system from reaching true equilibrium; the model may thus illustrate both nonequilibrium phase transitions (and critical phenomena) and their possible occurrence in disordered systems. We report in this note the main exact results from our study of this model; we hope this will inspire further interest in the model perhaps motivating, in particular, a related experiment. A more detailed mathematical study of a model which generalizes the present one is reported in López-Lacomba and Marro (1992).

Consider an interacting-spin d-dimensional lattice system at temperature T with configurations $s \equiv \{s_r = \pm 1\}$ whose probability at time t satisfies a (marcovian) master equation (Glauber, 1963; Kawasaki, 1972; Ligget, 1985), i.e., $\partial P(s;t)/\partial t = \Sigma_s[c(s\,|s^r)P(s^r;t) - c(s^r|s)P(s;t)]$, where s^r is the configuration obtained from s after flipping the spin at lattice site r, $s_r \to -s_r$. Unlike in more familiar cases, the transition rates per unit time for transitions from s^r to s, $c(s|s^r) \geq 0$, will involve here a series of simultaneously competing independent spin-flip or Glauber (1963) mechanisms. Namely,

$$c(s^r|s) = [\![c(s^r|s;h)]\!] \equiv \int_{-\infty}^{+\infty} dh f(h) c(s^r|s;h).$$

Here, h represents a random applied magnetic field which has a (normalized) distribution $f(h)$, and each elementary Glauber mechanism driven by $c(s^r|s;h)$ satisfies a detailed balance condition. Namely, $c(s^r|s;h) = c(s|s^r;h)\exp[-\beta\Delta H_n]$, with $\beta \equiv (k_B T)^{-1}$ and $\Delta H_h \equiv H(s^r;h) - H(s;h)$. This involves a series of "Hamiltonians", $H(s;h)$; these may be assumed to have the Ising structure, for example, i.e., $H(s;h) = -J\Sigma_{NN}s_r s_r - h\Sigma_r s_r$, for all h, where the first sum is over nearest-neighbour (NN) pairs of sites and J is a constant.

This model has two simple interpretations. On the one hand, each elementary Glauber (canonical) mechanism acts at each kinetic step as if the strength of the applied field is h all over the system, with h selected at random from $f(h)$. Disregarding possible experimental difficulties, this situation may be implemented in the laboratory by changing very rapidly the field acting on a magnetic sample, as indicated above. Alternatively, given that Glauber processes are local, one may interpret the model as one where only the field on the spin whose flip is attempted is changed at each kinetic step. Thus, independently of the initial distribution of local fields, the system will always be acted on by a *spatial* field distribution which is a realization of $f(h)$ after some transient time. This is similar to the situation in the QRFIM, except that such a spatial distribution will keep changing with time independently of the spin system. We claim that similar changes in the spatial distribution of local fields may occur in practice, for example, in a random-field system in which the magnetic ions (acted on by random local fields) diffuse randomly, or in a dilute antiferromagnet (in a uniform field) constantly having a random migration of non-magnetic ions. One may convince oneself that the above two interpretations imply the same thermodynamics, except for the definition of some fluctuations. For simplicity, we shall refer explicity in the following to the former case, i.e., to a magnetic system under a fluctuating field. The system has then two well-defined limits for $f(h) = \delta(h \pm h_0)$ corresponding to the (pure) kinetic Ising model with equilibrium states characterized by temperature T and energy $H(s; \pm h_0)$, respectively, whose nature is well known. Excluding these limits, the simultaneous competition between the independent fields at each kinetic step will in general induce, as if the system were acted on by some external non-Hamiltonian agent, an asymptotic tendency towards a steady non-equilibrium state which may depend on $f(h)$ and $c(s^r|s; h)$ in addition to T.

The physical situations of interest naturally suggest that one can assume that the rates can only depend on the "energy" cost of the attempted transition, e.g., $c(s^r|s; h) = \phi(\beta\Delta H_h)$, where $\phi(X)$ is an arbitrary function, except that $\phi(X) = e^{-X}\phi(-X)$ in order to fulfill the detailed balance condition. Lacking other criteria, we shall consider the case $\phi(X) = e^{-1/2X}[\cosh(\frac{1}{2}\alpha X)]^{-1}$. This is actually rather general, with $\alpha = 0, \pm 1$ corresponding to familar rates introduced before in different problems (see, for instance, Ligget, 1985; Kawasaki, 1966). We shall consider explicitly distributions

$$f(h) = \tfrac{1}{2}p\delta(h - \mu_+) + \tfrac{1}{2}p\delta(h - \mu_-) + (1 - p)\delta(h - \mu), \; \mu_\pm = \mu \pm \zeta, \tag{1}$$

which are characterized by $[\![h]\!] = \mu$ and $[\![h^2]\!] = \mu^2 + p\zeta^2$. This induces a rich behaviour, and it allows for a significant comparison with previous results by Grinstein and Mukamel (1983) for the one-dimensional QRFIM; in fact, these authors considered a field which is (spatially) distributed according to a particular case of (1).

Under the conditions stated, it follows by simple algebra that the stationary solution of the above master equation may generally be written for the one-dimensional system as* $P^{st}(s) = Z^{-1}\exp[-E(s)]$ with $Z \equiv \Sigma_s \exp[-E(s)]$ and

$$E(s) = -K_e\Sigma_r s_r s_{r+1} - \beta h_e \Sigma_r s_r, \tag{2}$$

* The details of this proof, as well as some general theorems for systems with competing kinetics, have been worked out in López-Lacomba, Garrido and Marro (1990).

where K_e and h_e depend on T, $f(h)$ and $c(s^r|s;h)$. Thus, even though a free energy is lacking here (i.e., one may think of (2) as describing the spins acted on by an external agent), the system properties may be obtained from Z. For instance, the magnetization is simply $M \equiv Z^{-1}\{\partial Z/\partial(\beta h_e)\}$, and one may define the energy as $U \equiv -J\langle\Sigma_r s_r s_{r+1}\rangle - \lambda\langle\Sigma_r s_r\rangle$, where $\lambda \equiv [\![h]\!] = \mu$; $\langle\cdot\rangle$ represents the average with respect to $P^{st}(s)$. Thus, $U = -J(\partial \ln Z/\partial K_e)\lambda[\partial \ln Z/\partial(\beta h_e)]$. The resulting behaviour may be summarized as follows.

When $\alpha = 0$, one gets $K_e = \beta J$, and $\tanh(\beta h_e) = [\![\sinh(\beta h)]\!][\![\cosh(\beta h)]\!]^{1}$ which implies $h_e = \mu$ for any symmetric distribution (1). Consequently, the system behaves the same as the (equilibrium) Ising model under a field μ, except for specific non-equilibrium fluctuations due to the additional disorder induced by the fluctuating field. Namely, there is no fluctuation-dissipation relation; the mean square fluctuations of the energy are instead $\sigma_u^2 \equiv \langle[\![[H(s;h) - U]^2]\!]\rangle = k_B T^2 C_v + \sigma_n^2$. Here, $C_v \equiv (\partial U/\partial T)_{\nu,...}$ represents a *specific heat*, and the extra fluctuations are $\sigma_n^2 = \sigma_h^2(\sigma_M^2 + M^2)$, where $\sigma_h^2 \equiv [\![(h-\mu)^2]\!] = p\zeta^2$. The mean square fluctuations of the magnetization, $M = N\omega\sinh(\beta\mu)$, are

$$\sigma_M^2 \equiv [\![(\Sigma_r s_r) - M]^2]\!] = N\omega\cosh(\beta\mu)\{1 - [\omega\sinh(\beta\mu)]^2\},$$

where $\omega \equiv [\sinh^2(\beta\mu) + e^{-4\beta J}]^{-1/2}$ and N is the lattice size. Thus, as long as $\mu = 0$, it follows that $\sigma_n^2 = pN\zeta^2 e^{2\beta J}$, and also the familiar behaviour $\sigma_U^2 U^{-2} \to 0$ as $N \to \infty$, while we have $\sigma_U^2 U^{-2} \sim p\zeta^2(M/U)^2$ in the same limit when $\mu \neq 0$. The latter fact implies, in particular, that, unlike for familiar cases, $\sigma_U^2 U^{-2}$ has a non-zero value as $T \to 0$ (when $\mu \neq 0$). The resulting critical exponents when $\mu = 0$, on the other hand, equal those of the Ising model. The relevance of these similarities between equilibrium and certain nonequilibrium situations should not be overestimated, however. In fact, they are a consequence of the symmetry of $f(h)$. For instance, the distribution $f(h) = p\delta(h - \mu_+) + (1-p)\delta(h - \mu_-)$, $\mu_+ > \mu_-$, produces instead $K_e = \beta J$ and $\tanh(\beta h_e) = [t_+ + (1-2p)t_-][1 - (1-2p)t_+ t_-]^{-1}$ with $t_\pm \equiv \tanh[\frac{1}{2}\beta(\mu_+ \pm \mu_-)]$.

The choice $\alpha = 0$, which fixes a given kind of kinetics, also plays a simplifying role in our system. In fact, one may prove that, for $\alpha \neq 0$, the restriction to even distributions, $f(h) = f(-h)$, is a necessary and sufficient condition for the existence of a solution (2) satisfying $E(s) - E(s^r) = \ln[c(s^r|s)/c(s|s^r)]$. Contrary to the situation above, it then follows that $h_e = 0$ and

$$\tanh(2K_e) = [\![\sinh[\beta(h+2J)]/\cosh[\alpha\beta(h+2J)]]\!]\{[\![\cosh[\beta(h+2J)]/\cosh[\alpha\beta(h+2J)]]\!]\}^{-1},$$

or $\tanh(2K_e) = [\![\tanh[\beta(h \pm 2J)]]\!]$ for the particular case $\alpha = \pm 1$. For a distribution of zero mean,

$$f(h) = \tfrac{1}{2}p\delta(h-\zeta) + \tfrac{1}{2}p\delta(h+\zeta) + (1-p)\delta(h), \tag{3}$$

we get $U = -2NJ\sin^2(K_e)[\sinh(2K_e)]^{-1}$, $C_v = -4NJ\sinh^2(K_e)[\sinh(2K_e)]^{-2}\cdot(\partial K_e/\partial T)$, $M = 0$, $\sigma_M^2 = N\exp(2K_e)$,

$$\sigma_U^2 = 4NJ^2\sinh^2(K_e)[\sinh(2K_e)]^{-2} + N\sigma_h^2\exp(2K_e)$$

and $\sigma_h^2 = p\zeta^2$, where

$$\tanh(2K_\alpha) = \tanh(2\beta J)[\cosh^2(2\beta J) + (1-p)\cdot\sinh^2(\beta\zeta)][\cosh^2(2\beta J) + \sinh^2(\beta\zeta)]^{-1}$$

for $\alpha = \pm 1$. The main conclusion which follows from these equations is that, unlike the standard QRFIM, the system here depicts an interesting, nonuniversal critical behaviour as $T \to 0^+$.

More specifically, the neighbourhood of the zero-T point varies essentially according to the relation between $2J$ and ζ. One has as $T \to 0$:

a) $K_e \sim K(1 - \zeta/2J) + \frac{1}{4}\ln(2/p)$ for $2J > \zeta$,

b) $K_e \sim \frac{1}{4}\ln[(4-p)/p]$ for $2J = \zeta$, and

c) $K_e \sim \frac{1}{4}\ln[(2-p)/p]$ for $2J < \zeta$.

Consequently, case **a** is characterized by $U \to -NJ$, $M = 0$, $C_V \to 0$, $\sigma_M^2 \sim N(2/p)^{1/2}\cdot e^{\beta(2J-\zeta)} \to \infty$ and $\sigma_U^2 \sim \sigma_h^2 N(2/p)^{1/2} e^{\beta(2J-\zeta)} \to \infty$. Case **b** is distinctly characterized by

$$U \to NJ\{1 - [(4-p)p^{-1}]^{1/2}\}\{1 + [(4-p)p^{-1}]^{1/2}\}^{-1},\ C_V \to 0,$$

$$\sigma_M^2 \to N[(4-p)p^{-1}]^{1/2},\ \sigma_h^2 = p\zeta^2 = 4pJ^2$$

and $\sigma_U^2 \to 2NJ^2\{1 - 4[2 + (p\{4-p\})^{1/2}]^{-1}\} + N\sigma_h^2[(4-p)p^{-1}]^{1/2}$, while it follows in case **c** that $U \to -NJ\{1 - [p(2-p)]^{1/2}\}[2(1-p)]^{-1}$, $C_V \to 0$, $\sigma_M^2 \to N[(2-p)p^{-1}]^{1/2}$ and $\sigma_U^2 \to 2NJ^2\{1 - 2[1 + (p\{2-p\})^{1/2}]^{-1}\} + N\sigma_h^2[(2-p)p^{-1}]^{1/2}$. Concerning critical behaviour, for instance, these equations imply that the "pure" zero-T critical point has been washed out in the latter two cases, as in the QRFIM, due to the strong randomness induced by the fields. There is, however, some very atypical situation in case **c** with $p = 1$, namely, $U \to 0$, $C_V \to 0$, $\sigma_U^2 \to N\sigma_h^2$, $\sigma_M^2 \to N$, and $K_e \to 0$ as $T \to 0$. It may also be noticed that the correlation length, which is defined as $\xi \equiv -[\ln\tanh(K_e)]^{-1}$, behaves like $\xi \sim \frac{1}{2}[(4-p)p^{-1}]^{1/2}$ and $\frac{1}{2}[(2-p)p^{-1}]^{1/2}$, respectively, for cases **b** and **c**. That is $\xi \sim p^{-1/2}$ as $p \to 0$ so that one may define a *non-thermal* critical exponent $\nu_p = \frac{1}{2}$. On the contrary, case **a** retains the (familiar) thermal critical point, though this has a nature which crucially differs from the one in the pure model, i.e., $\xi \sim (\frac{1}{8}p)^{1/2}\varepsilon^{-\nu}$, where $\varepsilon \equiv \exp(-2\beta J)$ and $\nu = 1 - \zeta/2J$.

The quenched case studied by Grinstein and Mukamel (1983) may be compared with the above one when $\zeta \to \infty$. In fact, a motivation for this model (GMM) was aroused by the relevance of the parameter ζ/J observed by these authors. It should be remarked, however, that even though the QRFIM may easily be shown to map onto the Mattis (1976) spin-glass model *under a field*, which is in turn equivalent to the more standard spin-glass model by Edwards and Anderson (1975) (*only*) when $d = 1$, there is no general exact solution for the one-dimensional QRFIM, i.e., a complete solution for arbitrary T and field distribution. Consequently, any comparison we may report needs to refer to rather fragmentary equilibrium results. We may compare, for instance, results for the spin-spin correlation function, $g(l) \equiv \langle s_R s_{R+l}\rangle_0$, where $\langle\cdot\rangle_0$ involves both the usual ensemble average and the disorder average with respect to $f(h)$. Grinstein and Mukamel have found that, for $z \equiv \tanh(\beta J) \neq 1$ ($T \neq 0$) and $lz^l \ll 1$ (which implies $T \to \infty$, in particular), one has $g(l) \sim (pl+1)(1-p)^l z^l$ neglecting terms of order $l(1-p)^l z^{3l}$ or smaller. In our model, $g(l) = [\tanh(K_e)]^l$. For

$\alpha = \pm 1$, a distribution (3), and large ζ (namely, for any $\zeta > 2J$), the latter result implies in this limit that $g(l) \sim (1-p)^l z^l$ neglecting terms of order $l(1-p)^l z^{l-2}$ or smaller. That is, the high-temperature nonequilibrium correlation shows a purely exponential behaviour (as is expected to occur in the present nonequilibrium case; see Cheng, Garrido, Lebowitz and Vallés, 1991, and references therein), unlike what apparently occurs in the GMM. The correlation length behaves the same way in both cases: $\xi^{-1} \approx -\ln z - \ln(1-p)$. When $T = 0$, $g(l) \sim \text{const}(1-p)^l$ in the GMM, and $g(l) \sim [\frac{1}{2}(1-p)]^l$ here. Again, a warning seems pertinent: the similarities when $d = 1$ between the QRFIM (or the GMM) and our model should not be extrapolated beyond the conditions stated above, e.g., it is obvious that the two models essentially differ when the kinetics corresponds to the case $\alpha = 0$, and a similar conclusion follows when implementing the rates by the Metropolis algorithm (Metropolis, Rosenbluth, Rosenbluth, Teller and Teller, 1953), for instance.

A more complete evaluation of the significance of our model compels one to compare this with the random-field system in which the impurities are not quenched but annealed, i.e., they have reached equilibrium with the other degrees of freedom instead of remaining frozen in. This may be defined via

$$Z_N = \langle Z_N(T, h) \rangle \equiv \int \prod_{j=1}^{N} dh_j f(h_j) \sum_s \exp[\beta \sum_i s_i(Js_{i+1} + h_i)].$$

It follows that the partition function Z_N may be written as $\{[\![\cosh(\beta h)]\!]^2 - [\![\sinh(\beta h)]\!]^2\}^{1/2N} \equiv \varkappa$ times that of the familiar NN one-dimensional Ising model under a field Λ, with

$$\Lambda = \tfrac{1}{2}\beta^{-1} \ln\{[[\![\cosh(\beta h)]\!] + [\![\sinh(\beta h)]\!]][[\![\cosh(\beta h)]\!] - [\![\sinh(\beta h)]\!]]^{-1}\}. \tag{4}$$

Thus, given that κ is independent of s and that (4) leads to

$$\tanh(\beta\Lambda) = [\![\sinh(\beta h)]\!][[\![\cosh(\beta h)]\!]]^{-1},$$

the solution $P^{st}_{ann}(s)$ implied by Z_N equals the one for the Ising model under a field Λ, and also the one for the nonequilibrium system with $\alpha = 0$. Consequently, annealed configurational quantities such as the magnetization, its fluctuations, and spin-spin correlation functions are identical to those for the two above-mentioned systems, while thermal quantities such as U, C_V, etc. will in general differ essentially, and will show up a strong dependence on $f(h)$ in addition to the one involved by Λ. For example, when one defines ΔU as the difference between the energies for the nonequilibrium (with $\alpha = 0$) and annealed systems, one gets $\Delta U = \zeta\tanh(\beta\zeta)$ for distribution (1) when $p = 1$; thus, $\Delta U \to 0$ as $T \to \infty$, and $\Delta U \to \zeta$ as $T \to 0$. In any case, the comment at the end of the last paragraph also applies here; in fact, the disorder in our system is not in equilibrium with the other degrees of freedom.

Some exact results concerning the nonequilibrium system with arbitrary d may also be pointed out. The fact that transition rates need to have a positive lower bound (Ligget, 1985) implies that, for any distribution $f(h)$ and $\alpha = 0$, for instance, the system is ergodic (so that it needs to remain in a unique phase) when $\beta < \beta_0$, where the latter satisfies $[1 + \tanh(\beta_0 J)]^{2d} = 2\{1 + [[\![\sinh(\beta_0 h)]\!][\![\cosh(\beta_0 h)]\!]]\}^{-1}$. This defines a region of the phase diagram where long-range order may be found. Similar results follow easily for other rates. It is also worthwhile to note that the model behaviour may be represented at $T = 0$ by simple random cellular automata. For instance, the case $\alpha = \pm 1$ with distribution (3) is equivalent at $T = 0$ to the cellular automaton $c(s^r|s) = 1 - s_r\sigma_1[1 + \frac{1}{2}p\theta(\lambda_1)]$, $1 - s_r[x(\sigma_1 + \sigma_2) + y(\sigma_1\tau_2 + \sigma_2\tau_1)]$ and

$$1 - s_r[z\Sigma_l\sigma_l + w(\tfrac{1}{2}\Sigma_{l,m}\sigma_l\tau_m + \sigma_1\sigma_2\sigma_3) + v\Sigma_{l,m,n}\sigma_l\tau_m\tau_n]$$

(with $l, m, n = 1, 2, 3$, $l \neq m \neq n$) for $d = 1, 2$ and 3, respectively. The following notation has been used to shorten the formulae: $\sigma_1 \equiv \frac{1}{2}(s_{r+i} + s_{r-i})$, $\sigma_2 \equiv \frac{1}{2}(s_{r+j} + s_{r-j})$, $\sigma_3 \equiv \frac{1}{2}(s_{r+k} + s_{r-k})$, $\tau_1 \equiv s_{r+i}s_{r-i}$, $\tau_2 \equiv s_{r+j}s_{r-j}$, and $\tau_3 \equiv s_{r+k}s_{r-k}$, where $r \pm r'$ ($r' = i, j$ or k) represent NN of site r,

$$x \equiv \tfrac{1}{8}\{4 + p[2 + 2\theta(\lambda_1) + \theta(\lambda_2)]\},$$
$$y \equiv \tfrac{1}{8}[-2 - 2p\theta(\lambda_1) + p\theta(\lambda_2)],$$
$$z \equiv \tfrac{1}{8}\{4 + \tfrac{1}{4}p[3\theta(\lambda_1) + 4\theta(\lambda_2) + \theta(\lambda_3)]\},$$
$$w = \tfrac{1}{8}[-4 - 3p\theta(\lambda_1) + p\theta(\lambda_3)],$$

and

$$v = \tfrac{1}{8}\{1 + \tfrac{1}{4}p[5\theta(\lambda_1) - 4\theta(\lambda_2) + \theta(\lambda_3)]\},$$

$\theta(X) \equiv \Theta(X)[\Theta(-X) - 2]$ where $\Theta(X) = 0$ for $X < 0$ and $\Theta(X) = 1$ for $X \geq 1$, and $\lambda_n \equiv \zeta/2J - n$. This algorithm may easily be implemented on a computer to obtain information about the behaviour of the system at very low temperatures. Moreover, the positivity of the lower bound for $c(s^r|s)$ characterizing this cellular automaton may be seen to imply, in particular, that the system at zero temperature with $\zeta > 6J$ is ergodic, so that no phase transition can occur, for any $p > 0$ when $d = 1$ for any $p > \frac{1}{3}$ when $d = 2$, and for any $p > \frac{23}{31}$ when $d = 3$.

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References

Belanger, D. P., S. M. Rezende, A. R. King and V. Jacarino. (1985). Hysteresis, metastability and time dependence in $d = 2$ and $d = 3$. *J. Appl. Phys.*, **57**, 3294.

Birgeneau, R. J., Y. Shapira, G. Shirane, R. A. Cowley and H. Yoshizawa. (1986). Random fields and phase transitions. *Physica B*, **137**, 83.

Bricmont, J. and A. Kupiainen. (1988). Phase transitions in the 3d random field Ising model. *Commun. Math. Phys.*, **116**, 870.

Bruinsma R. and G. Aeppli. (1983). One dimensional Ising model in a random field. *Phys. Rev. Lett.*, **50**, 1494.

Cheng, Z., P. L. Garrido, J. L. Lebowitz and J. L. Vallés. (1991). Long range correlations in stationary nonequilibrium systems with conservative anisotropic dynamics. *Europhys. Lett.*, **14**, 507.

Derrida, B., J. Vannimenus and Y. Pomeau. (1978). Simple frustrated systems: chains, strips and squares. *J. Phys.*, **C11**, 4749.

Edwards, S. F. and P. W. Anderson. (1975). Theory of spin glasses. *J. Phys.*, *F* **5**, 965.

Fishman, S. and A. Aharony. (1979). Random field effects in disordered anisotropic antiferromagnets. *J. Phys.*, *C* **12**, L729.

Garrido, P. L. and J. Marro. (1991). A nonequilibrium version of the spin-glass problem. *Europhys. Lett.*, **15**, 375.

Garrido, P. L. and J. Marro. (1992). Nonequilibrium impure lattice systems. *J. Phys.*, *A* **25**, 1453.

Glauber, R. J. (1963). Time dependent statistics of the Ising model. *J. Math. Phys.*, **4**, 294.

Grinstein, G. and D. Mukamel. (1983). Exact solution of a one-dimensional Ising model in a random magnetic field. *Phys. Rev.*, *B* **27**, 4503.

Imry, Y. and S. K. Ma. (1975). Random field instability of the ordered state of continuous symmetry. *Phys. Rev. Lett.*, **35**, 1399.

Imry, Y. (1984). Random external fields. *J. Stat. Phys.*, **34**, 849.

Kawasaki, K. (1966). Diffusion constants near the critical point for time dependent Ising models: I. *Phys. Rev.*, **145**, 224.

Kawasaki, K. (1972). Kinetics of Ising models. In C. Domb and M. S. Green (Eds.), *Phase Transitions and Critical Phenomena*, Vol. 4, Academic Press, London 1972.

Ligget, T. M. (1985). *Interacting Particle Systems*. Springer-Verlag, Berlin.

López-Lacomba, A. I. and J. Marro (1992). Nonequilibrium phase transitions in lattice systems with a random-field competing kinetics. *Phys. Rev. B*, **46**, in press.

López-Lacomba, A. I., P. L. Garrido and J. Marro. (1990). Stationary distributions for systems with competing creation-annihilation dynamics. *J. Phys.*, *A* **23**, 3809.

Mattis D. (1976). Solvable spin systems with random interactions. *Phys. Lett.*, *A* **56**, 421.

Metropolis, N., A. W. Rosenbluth, N. M. Rosenbluth, A. H. Teller and E. Teller. (1953). Equation of state calculations by fast computing machines. *J. Chem. Phys.*, **21**, 1087.

Natterman, T. N. and J. Villain. (1988). Random-field Ising systems: a survey of current theoretical views. *Phase Transitions*, **11**, 5.

Wehr, J. (1989). Ph.D. Thesis (unpublished). Rutgers University, New Jersey.