

Non-equilibrium impure lattice systems*

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Abstract. We study d -dimensional lattice model systems whose dynamics consist of competing independent processes which may induce non-equilibrium steady states. Interesting phase transitions and critical phenomena ensue which are influenced, even dominated, by a sort of *dynamical* bond disorder. The latter, which may occur in natural systems, reminds us of the disorder in more familiar bond-diluted and other impure models, for example percolation-like phenomena arise in some of our model cases. Exact solutions for $d = 1$, some exact results for $d > 1$, and a comparison with more standard, either quenched or annealed, magnetically diluted models are reported.

1. Introduction and definition of systems

Lattice systems with quenched (frozen-in) impurities nowadays have an undeniable interest, both as an ideal representation of some disordered systems in Nature and as a convenient arena for studying critical phenomena one step beyond the familiar pure models. A well defined case which has been attracting considerable attention is the randomly dilute quenched Ising model whose impurities are either non-magnetic sites or broken bonds. Pioneering studies revealed that the transition temperature for Ising ferromagnets with quenched dilution decreases monotonically from its pure value to zero at the percolation threshold, and the system is characterized by percolation critical behaviour as one varies the dilution parameter at zero temperature (see, for instance, Stauffer 1979). More recently, Monte Carlo and other techniques have revealed that the transition for a 3D lattice with quenched site dilution is a sharp second-order one with *effective* critical exponents changing from the Ising pure value to a new set of impure values. This may be interpreted as caused by a flow from the (unstable) pure fixed point to a stable impure fixed point, the latter associated with a very small asymptotic critical region. This seems to be so small that it sometimes prevents one from detecting impure behaviour while, most often, it leads in practice to a monotonic variation of the critical exponents with dilution (Marro *et al* 1986, Labarta *et al* 1986). Although this picture has essentially been confirmed by several other independent studies (Chowdhury and Stauffer 1986, Braun and Fähnle 1988, Holey and Fähnle 1990, Heuer 1990, and references therein), the relative smallness of the impure critical

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region and other facts have so far precluded the definite exclusion (see, however, Wang and Chowdhury 1989) of the case of a line of fixed points, one for each value of the impurity concentration. Even though this would indeed be a rare situation in equilibrium phenomena, one should conclude that, in spite of much recent effort, the magnetically diluted problem is still open today. In fact, it may even be said that the actual relevance of that (quenched) bond-impure Ising model for interpreting the properties of a class of magnetic materials remains unclear.

We shall here try to approach the understanding of that problem from a novel point of view. We investigate the behaviour of a d -dimensional nearest-neighbour (NN) magnetic Ising model whose dilution (actually, a sort of bond dilution) is caused by a relatively complex kinetic process which also induces (in general) the presence of non-equilibrium steady states. Namely, our system has a Markovian time evolution due to the simultaneous competition of (e.g.) two independent local spin-flip processes, each having a different probability, in addition to the action of a thermal bath. Those two processes occur as if the exchanged interaction (*bond*) between the involved pairs of spins has two different values, J and 0 respectively. This asymptotically produces a random spatial distribution of broken bonds which is similar *at each time* to the one in the quenched dilute system. The motivation for this model is threefold. First, while *solvable* stochastic microscopic models capturing essential features of disordered systems are scarce at present, one is confronted here with model systems whose exact solution is feasible for $d = 1$; moreover, some exact results may also be obtained for $d > 1$. Second, the chances are that, due to atomic migration, the macroscopic properties of magnetically diluted materials are *more closely due to an impurity distribution* which changes with time (perhaps in some way resembling the situation in our model) than to the quenched distribution in the more familiar models. Finally, our models lead to the study of *non-equilibrium* steady states, phase transitions and critical phenomena which is an active area nowadays (see, for instance, Onuki and Kawasaki 1981, Katz *et al* 1984, van Beijeren and Schulman 1984, Marro 1986, Marro *et al* 1987, Dickman 1989, Droz *et al* 1989, Haider *et al* 1990, Garrido *et al* 1990). In fact, a conclusion there is that critical phenomena in non-equilibrium systems involving disorder are amazingly rich. Our models may thus allow for stringent tests of concepts and theories.

The same philosophy may of course be applied to the study of other impure systems. For instance, in relation to *frustration* (see, for instance, Fisher *et al* 1988 and references therein), it is rewarding to consider an Ising system whose impurities are spins at one of the two possible values, either $+1$ or -1 , in such a way that they introduce no net magnetization. The Monte Carlo study of that system in three dimensions (Labarta *et al* 1988) reveals a situation essentially differing from that described above for the dilute Ising case. Namely, one finds first-order phase transitions, and also second-order ones (the latter seem to be characterized by small values of the magnetization critical exponent, thus sometimes resembling first-order transitions); the nature of the phase transition essentially depends on the symmetries of the impurity distribution. As a further step towards a better understanding of those impure situations, we investigate here a ferromagnetic model in which competing kinetics simulate the existence, with a given probability, of antiferromagnetic bonds having a very large strength. Of course, *other impure situations may in principle* be investigated by following a similar strategy, as in two recent studies concerning non-equilibrium spin-glass (Garrido and Marro 1991a) and random-field (López-Lacomba *et al* 1991) systems, respectively. Eventually, we also refer in this paper to those non-equilibrium versions of familiar disordered

models because they are intimately related to those we are studying here. The application of our strategy to the study of dilute spin glasses (Klein *et al* 1989, and references therein) also seems interesting *a priori*.

This paper is organized as follows. The basic model system is defined in the remainder of section 1. Section 2 introduces notation, discusses transition rates, and contains a brief description of some techniques and general results which are necessary in our study. Sections 3 and 4 are devoted to the solution of the two main 1D versions of interest, respectively. In particular, we also introduce in section 3 a proper definition of energy. Then, we compare in section 5 our model systems with the corresponding, more familiar, equilibrium ones having either quenched or annealed impurities. Section 6 contains some exact results concerning 2D and 3D systems, namely upper bounds for transition temperatures in various situations and cellular automata representations of the systems at $T=0$. Section 7 summarizes our main conclusions.

Consider a d -dimensional lattice Ω which is in contact with a thermal bath at temperature T . The probability of any configuration $s \equiv \{s_x = \pm 1; x \in \Omega\}$ at time t satisfies the (Markovian) master equation (see, for instance, Liggett 1985):

$$\partial P(s; t) / \partial t = \sum_x [\omega(s^x; x) P(s^x; t) - \omega(s; x) P(s; t)]. \tag{1.1}$$

This describes stochastic changes $s_x \rightarrow -s_x$ of the spin (or particle occupation) variable at site x which generate a new configuration, to be denoted s^x , from s with probability $\omega(s; x)$ per unit time. Unlike in the familiar Glauber (1963) case, however, the microscopic dynamics here are assumed (see also Marro and Garrido 1990) to involve n competing Glauber spin-flip (or creation-annihilation) mechanisms. That is,

$$\omega(s; x) = \sum_i p_i \omega_i(s; x) \quad \sum_i p_i = 1 \quad (i = 1, \dots, n). \tag{1.2}$$

It is convenient to assume that each elementary kinetic mechanism driven by ω_i satisfies a detailed balance condition, namely

$$\omega_i(s; x) = \omega_i(s^x; x) \exp[-\Delta H_i] \quad \Delta H_i \equiv H_i(s^x) - H_i(s) \tag{1.3}$$

with respect to some specific 'Hamiltonian'. We shall assume, for simplicity,

$$H_i(s) = -K_i \sum_{NN} s_x s_{x'} - \beta h \sum_x s_x \quad K_i \equiv \beta J_i \quad \beta \equiv (k_B T)^{-1} \tag{1.4}$$

where the first sum is over all NN pairs of sites.

The interpretation of (1.1)-(1.4) is that kinetics involve several (canonical) Glauber mechanisms, each acting with probability p_i as if the strength of the bonds has a given value J_i . In other words, our system is precisely the Glauber (1963) or kinetic Ising model with non-conserved magnetization (see, for instance, Stanley 1971), except that the value of J implied at each kinetic step is chosen at random from some given distribution $p(J)$. Given that the kinetical processes of interest are local (e.g. (1.2) with (1.3) and (1.4) only involve a given spin, say s_x , and its NN) one may interpret alternatively either that J is changed (according to $p(J)$) at each step all over the system, or else that the change only affects the bonds reaching s_x . When $p(J) = \delta(J - J_0)$, where J_0 is a constant and δ represents a Dirac delta function, any spin-flip rate satisfying (1.3) drives the system to the Gibbs equilibrium state corresponding to temperature T and energy (1.4), with $K_i = \beta J_0$ for all i , whose nature is well known. For example, the system undergoes a continuous phase transition at critical temperature $T_c \geq 0$ for $d \geq 1$, respectively. Under general conditions, however, the situation is more

complex. Namely, the competition between several values of J will in general cause the bonds to be randomly distributed (always according to $p(J)$), either only in time or in time and spatially at each time, respectively, for those two interpretations. That competition will thus induce two main effects:

- (i) a completely random distribution of the bond disorder, as if it is produced by some mechanism at *infinite temperature*;
- (ii) a tendency of the spin system towards a non-equilibrium steady state, as if it is acted on by some external agent which is non-Hamiltonian in general.

Under those conditions, we would like to understand the dependence of the non-equilibrium state on $p(J)$, T , h and $\omega(s; x)$, analyse the nature of the phase transitions and critical phenomena which may possibly have the system, and investigate the suspected relevance of the model to understanding certain peculiarities of some disordered natural systems. In particular, we are interested here in distributions $p(J)$ such that the resulting system bears some interest in relation to the study of percolation-like phenomena and other *impure* situations, as explained above.

2. A quasicanonical competing kinetics

Sections 2-5 deal with $d = 1$. Then the most general transition rate having a local nature (i.e. it only involves sites which are NN of the one whose change is attempted, say s_x) which satisfies (1.3) with (1.4) is

$$\omega_i(s; x) = f_i^x(s; K_i)[1 - s_x\alpha_h + \alpha_i(\alpha_h - s_x)\sigma_1^x] \tag{2.1}$$

where $\alpha_i \equiv \tanh(2K_i)$, $\alpha_h \equiv \tanh(\beta h)$, $\sigma_1^x \equiv \frac{1}{2}(s_{x+1} + s_{x-1})$, $f_i^x(s; K_i) = f_i^x(s^x; K_i)$ and

$$f^x(s; K_i) = a_i + b_i\sigma_1^x + c_i\sigma_2^x \tag{2.2}$$

with $\sigma_2^x \equiv s_{x+1}s_{x-1}$. Here, $a_i = a(K_i)$, $b_i = b(K_i)$, $c_i = c(K_i)$, $a_i, b_i, c_i \in \mathbb{R}$ and $a_i > 0$, for all i , and we are assuming for simplicity that the functions $f_i^x(s; K_i)$ are the same for each individual rate ω_i , i.e. $f_i^x(s; K_i) = f^x(s; K_i)$ for all i . It thus follows that the global transition rate (1.2) may be written rather generally as

$$\omega(s; x) = A^x + B^x\sigma_1^x + C^x\sigma_2^x \tag{2.3}$$

where

$$A^x = \langle\langle a \rangle\rangle + \frac{1}{2}\alpha_h\langle\langle b\alpha \rangle\rangle - s_x[\alpha_h\langle\langle a \rangle\rangle + \frac{1}{2}\langle\langle b\alpha \rangle\rangle] \tag{2.4a}$$

$$B^x = \alpha_h\langle\langle a\alpha \rangle\rangle + \langle\langle b \rangle\rangle + \alpha_h\langle\langle c\alpha \rangle\rangle - s_x[\langle\langle a\alpha \rangle\rangle + \alpha_h\langle\langle b \rangle\rangle + \langle\langle c\alpha \rangle\rangle] \tag{2.4b}$$

$$C^x = \langle\langle c \rangle\rangle + \frac{1}{2}\alpha_h\langle\langle b\alpha \rangle\rangle - s_x[\alpha_h\langle\langle c \rangle\rangle + \frac{1}{2}\langle\langle b\alpha \rangle\rangle]. \tag{2.4c}$$

Here, $\alpha = \tanh(2K)$ and $\langle\langle \dots \rangle\rangle = \sum_i p_i \dots = \int dJ p(J) \dots$ where it is assumed that the distribution $p(J)$ is normalized to unity.

We may now apply to the present case some general concepts and theorems developed earlier [López-Lacomba *et al* 1990]. With that aim, one realizes that the stationary solution of (1.1), defined as the limit of $P(s; t)$ as $t \rightarrow \infty$, may be written as

$$P^{st}(s) = Z^{-1} \exp\{-E(s)\} \quad Z = \sum_s \exp\{-E(s)\}. \tag{2.5}$$

Consequently,

$$E(s) = \sum_{k=1}^N \sum'_{(x_1 \dots x_k)} J_{x_1 \dots x_k}^{(k)} s_{x_1} \dots s_{x_k} \tag{2.6}$$

where Σ' sums over every set of k lattice sites. Let us assume that

$$J_{x_1, \dots, x_k}^{(k)} = 0 \quad \text{for all } k \geq k_0 \tag{2.7}$$

where k_0 is independent of N , at least for $N > N_0 < \infty$. Thus, $E(s)$ has a short-range nature which guarantees it involves no infinite coefficients $J^{(k)}$ when it refers to a macroscopic ($N \rightarrow \infty$) system. When a unique stationary distribution function $P^{st}(s)$ exists such that (2.5)–(2.7) hold, $E(s)$ plays the role of an *effective Hamiltonian* (Garrido and Marro 1989). Previous theorems (López-Lacomba *et al* 1990) then allow us to conclude that, in so far as the rates satisfy

$$\langle\langle a \rangle\rangle + \langle\langle c \rangle\rangle \langle\langle b\alpha \rangle\rangle = (\langle\langle a\alpha \rangle\rangle + \langle\langle c\alpha \rangle\rangle) \langle\langle b \rangle\rangle \tag{2.8}$$

our system in section 1 has an effective Hamiltonian which is given by

$$E(s) = -K_e \sum_x s_x s_{x+1} - \beta h \sum_x s_x \tag{2.9}$$

where

$$K_e = -\frac{1}{8} \ln \{ [\langle\langle A(1-\alpha) \rangle\rangle \langle\langle B(1-\alpha) \rangle\rangle] [\langle\langle A(1+\alpha) \rangle\rangle \langle\langle B(1+\alpha) \rangle\rangle]^{-1} \} \tag{2.10}$$

with $A \equiv a + b + c$ and $B \equiv a - b + c$. That is, the effective Hamiltonian conserves in the present case the relatively simple structure of the original NN Ising Hamiltonian, (1.4), in so far as (2.8) is satisfied, while the ensuing coupling constant K_e involves a rather complex interplay of T , $p(J)$ and kinetics.

Note that the description contained in (2.5), (2.9) and (2.10) has a canonical structure, but it involves details of kinetics in addition to the disorder distribution. This may be interpreted by assuming that the configuration s is influenced, even dominated, by some imaginary external constraint which replaces the effect of the competing kinetics. That is, even though the spin system is relatively simple when (2.8) holds, it seems it still needs to be considered as a non-equilibrium system acted on by non-Hamiltonian agents. In any case, it is certainly exceptional that our model satisfies (2.8), as indicated in an example below, given that this follows from the requirement (López-Lacomba *et al* 1990) that the *effective* transition rate (1.2) satisfies a sort of global detailed balance condition with respect to $E(s)$, namely $\omega(s; \mathbf{x}) \exp[-E(s)] = \omega(s^x; \mathbf{x}) \exp[-E(s^x)]$.

Lacking other criteria, our transition rates may be those used before in different problems, for example by Glauber (1963), which correspond to (2.2) with

$$a = 1 \quad b = 0 \quad c = 0 \tag{2.11}$$

by Kawasaki (1972), i.e.

$$a = \frac{1}{2}(1 + \varepsilon^{-1}) \quad b = -(1 - \varepsilon)^{1/2} \varepsilon^{-1} \quad c = -\frac{1}{2}(1 - \varepsilon^{-1}) \tag{2.12}$$

where $\varepsilon \equiv 1 - \tanh^2(2K) \tanh^2(\beta h)$, or by van Beijeren and Schulman (1984), i.e.

$$a = \cosh(\beta h) \cosh^2(K) \quad b = 0 \quad c = \cosh(\beta h) \sinh^2(K). \tag{2.13}$$

Also interesting *a priori* are the rates characterizing the algorithm by Metropolis *et al* (1953) which correspond to

$$a = (1 + \mu)[(1 + \pi)^2 + 4\pi]/16\pi \quad b = -(1 + \mu)(1 - \pi^2)/8\pi \quad c = -\frac{1}{2}b \tag{2.14a}$$

when $h > 2J$, and to

$$\begin{aligned} a &= (1 + \mu)^2(1 + \pi)/16\mu + \frac{1}{4}(1 + \mu) \\ b &= -(1 + \pi)(1 - \mu^2)/8\mu \\ c &= (1 + \mu)^2(1 + \pi)/16\mu - \frac{1}{4}(1 + \mu) \end{aligned} \quad (2.14b)$$

when $h < 2J$; here, $\pi \equiv \exp(-4|K|)$, $\mu \equiv \exp(-2\beta h)$. Alternatively, we may consider the family of rates which depend only on the cost of the involved (spin-flip) transition. More precisely, it may be convenient to consider $\omega_i(s; x) = \phi(\Delta H_i)$, where $\phi(X)$ is any function satisfying $\phi(X) = e^{-X}\phi(-X)$, in order to fulfil condition (1.3), and $\phi(0) = 1$ and $\phi(X) \rightarrow 0$ as $X \rightarrow \infty$, in order to be properly normalized. Then, the rates by Glauber (1963) and by Kawasaki (1972) correspond to $\phi(X) = 1 - \tanh(\frac{1}{2}X)$, that by van Beijeren and Schulman (1984) is for $\phi(X) = e^{-X/2}$, and $\phi(X) = \min(1, e^{-X})$ represents the rate by Metropolis *et al* (1953), if, for simplicity, one assumes that no field exists ($h = 0$). The relation between the two representations (when $h = 0$) is

$$a, c = \frac{1}{2}[\phi(4K)(1 - \alpha)^{-1} \pm \phi(0)] \quad b = 0 \quad (2.15)$$

which defines parameters a , b and c satisfying condition (2.8). Thus, it follows after using (2.15) that (2.10) may simply be written as

$$K_e = -\frac{1}{4} \ln[\langle\langle \phi(4K) \rangle\rangle \langle\langle \phi(-4K) \rangle\rangle^{-1}]. \quad (2.16)$$

It should be emphasized that the sufficient condition (2.8) is not verified on most occasions. For instance, when $a = 1$, $b = -\tanh(2K) \tanh(\beta h)$ and $c = 0$, (2.8) reduces to $\langle\langle \alpha^2 \rangle\rangle = \langle\langle \alpha \rangle\rangle^2$ for $h \neq 0$ which does not hold unless $p(J)$ has zero variance, which occurs at equilibrium. This has two important consequences. On one hand, most versions of our system, particularly those for $d > 1$ (and/or $h \neq 0$), cannot be described by the *quasicanonical* formalism (2.5) + (2.9). In the light of the results in this paper, this suggests a rich and interesting behaviour for those cases (whose study is thus strongly encouraged). On the other hand, one should be able to obtain relevant information about those complex cases by a perturbative treatment around a reference situation which is characterized by the relatively simple quasicanonical formalism. For example, the system with $d = 1$, $h \neq 0$ and arbitrary $p(J)$ may be studied (e.g. to obtain the behaviour of the magnetic susceptibility) by performing an expansion around the corresponding case with $h = 0$.

It also seems worth while to emphasize that our basic model is defined via the Markovian equation (1.1) with rates (1.2). In a Monte Carlo study, one may either compute and use $\omega(s; x)$ as an effective rate, or perform the sampling which is suggested by (1.2) and then use the selected elementary rate $\omega_i(s; x)$ to decide on the attempted flip. It is not granted *a priori* that these two procedures should lead to the same steady state in the present non-equilibrium problem, however.

3. Non-magnetic impure bonds

Consider the distribution $p(J) = (1 - q)\delta(J - J_0) + q\delta(J)$: the bonds have strength $J = J_0$ in general, but are allowed occasionally to become zero with probability q . In addition to our general interest in the nature of the ensuing steady states, this may be of some relevance to understanding the influence of microscopic disorder on macroscopic properties, for example the problem in magnetically diluted systems. As a matter of fact, the present system will undergo a kind of percolation phenomena, even though

it essentially differs from the familiar (bond) diluted Ising model (cf section 5). Note also that the existence of an effective Hamiltonian in this case implies that most properties simply follow from $u \equiv \tanh K_e$ where $K_e = -\frac{1}{4} \ln(\alpha_+/\alpha_-)$, with $\alpha_{\pm} \equiv q + (1-q)\phi(\pm 4K_0)$ and $K_0 \equiv \beta J_0$, for the mentioned distribution; see figure 1 for a representation of this function.

When the rate function is analytical, one has for small X that $\phi(X) \approx 1 + X\phi'(0) + \dots$ with $\phi'(0) = -\frac{1}{2}$. Thus, $K_e \approx (1-q)K_0 + \dots$ in the limit $K_0 \rightarrow 0$. The latter is still valid for the whole family of rates we defined in section 2, even though (2.14) is not differentiable. On the other hand, one gets $K_e \approx -\frac{1}{2} \ln\{q + (1-q)\phi(-4K_0)\}^{-1}$ when $K_0 \rightarrow \infty$, and it is convenient to distinguish two different types of kinetics within that family, namely:

Case (i), which is characterized by the asymptotic property: $\phi(-X) \approx \mu > 0$ when $X \rightarrow \infty$, as for rates (2.11) and (2.14) with $\mu = 1$ and 2 respectively; and

Case (ii), which is characterized by $\phi(-X) \approx \exp(\pi X)$ (with $\pi < 1$ as required by detailed balance) when $X \rightarrow \infty$, as for rate (2.13) with $\pi = \frac{1}{2}$.

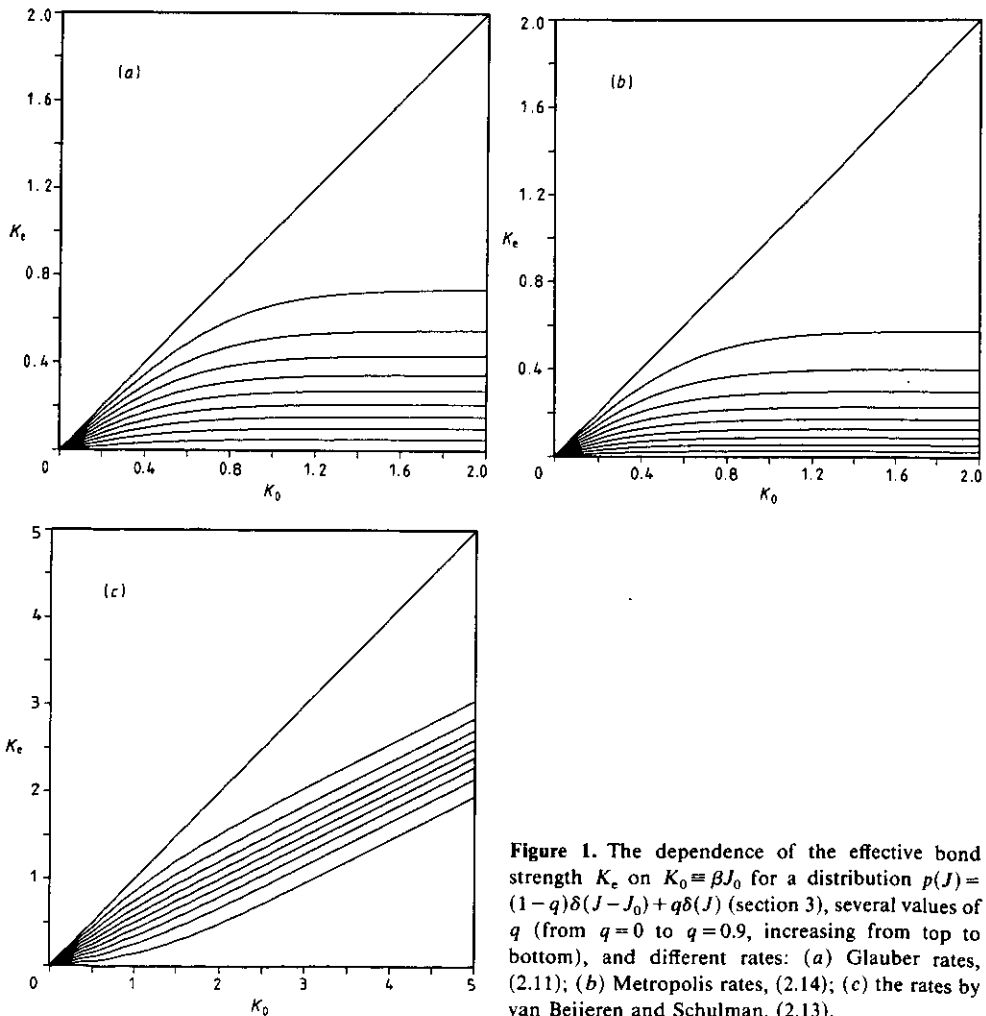


Figure 1. The dependence of the effective bond strength K_e on $K_0 \equiv \beta J_0$ for a distribution $p(J) = (1-q)\delta(J-J_0) + q\delta(J)$ (section 3), several values of q (from $q=0$ to $q=0.9$, increasing from top to bottom), and different rates: (a) Glauber rates, (2.11); (b) Metropolis rates, (2.14); (c) the rates by van Beijeren and Schulman, (2.13).

Interestingly enough, one gets $K_c \approx -\frac{1}{4} \ln\{q[q + (1-q)\mu]^{-1}\}$ as $K_0 \rightarrow \infty$ for case (i); consequently, K_c may only diverge for $q = 1$ (cf figures 1(a) and (b)), which reveals that any competition of that kind makes the standard pure (equilibrium) zero-temperature critical point disappear. This is similar to equilibrium where a 1D system will lose the critical point for any non-zero concentration of non-magnetic impurities (cf section 5). In that sense, case (ii) is rather novel given that $K_c \approx \pi K_0$ when $q \neq 0, 1$ as $K_0 \rightarrow \infty$, which reveals that the critical point is stable for that competition (cf figure 1c). The (thermal) critical exponents in the latter case, however, will equal π times the equilibrium ones, so that the neighbourhood of the critical point in parameter space strongly depends on kinetics.

The influence of kinetics on macroscopic properties is a distinguished property of non-equilibrium situations. It is noteworthy, for instance, that one may define a kind of kinetics for which the system loses the (equilibrium) critical point, while another class of kinetics exists, for example, one consisting of a competition of rates (2.13), which produces critical behaviour even for $q \neq 0$. One may argue that, for the bond distribution considered in this section, the disorder is introduced in the system on a time scale which is proportional to q^{-1} , while the order occurs on a time scale which is proportional to $[(1-q)\phi(-4K_0)]^{-1}$. Consequently, the condition

$$(1-q)\phi(-4K_0) \gg q \quad (3.1)$$

will guarantee the existence of a critical point. One may easily check how this occurs indeed for any case (ii) kinetics, while both time scales are comparable for any case (i) kinetics. One may also note that, as depicted by figure 1(c), case (ii) is characterized by a change-over at $q = \frac{1}{2}$ in the curvature of the K_c versus K_0 function which implies further qualitative changes of behaviour.

3.1. Correlation length

The spin-spin correlation function, $\langle s_0 s_x \rangle$, and the correlation length, ξ , follow from $\langle s_0 s_x \rangle = e^{-x/\xi} = u^x$. Consequently, $\xi \approx -(\ln K_0)^{-1}$ as $K_0 \rightarrow 0$ for $q \neq 1$, indicating that any kinetics behaves similarly and independently of q at high enough T . When $K_0 \rightarrow \infty$, however, the system may again show two different behaviours. Namely, for case (i), we find $\xi = -(\ln u)^{-1}$, with $u = (A - q^{1/2})(A + q^{1/2})^{-1}$ and $A \equiv [q(1-q)\mu]^{1/2}$, leading as $q \rightarrow 0$ to a divergence, $\xi \approx \frac{1}{2}\mu^{1/2}q^{-1/2}$, which is characterized by the percolation critical exponent $\nu_q = \frac{1}{2}$. Note that, as far as $\mu \neq 0$, this critical behaviour is independent of μ , i.e. of kinetics. For case (ii), however, $u \approx 1 - 2e^{-2\pi K_0}$ and one gets $\xi \approx \frac{1}{2}e^{2\pi K_0}$ and $\nu = 2\pi$, for example $\nu = 1$ for rates (2.13). This means that the correlation length exponent ν is not universal; more precisely, it may depend on asymptotic properties of kinetics, for example on the behaviour of $\phi(-X)$ as $X \rightarrow \infty$, but it is independent of q and other features. It may also be mentioned that, for a 1D pure system at equilibrium ($q = 0$), one has $\xi_0 \approx \frac{1}{2}e^{2K_0}$ implying $\nu = 2$, and $\langle s_0 s_x \rangle_q = (1-q)^x \langle s_0 s_x \rangle_0$ and $\xi_q^{-1} = -\ln(1-q) + \xi_0^{-1}$ when the system has bond dilution. Summing up, case (i) has some qualitative properties which are similar to those at equilibrium (e.g. the zero- T critical point washes out in the presence of any disorder, and percolation critical behaviour is characterized by $\nu = \text{constant}$), while case (ii) differs even more strongly, for example the disorder does not destroy the critical point and both thermal and percolation exponents depend on kinetics. Figure 2 illustrates some of these differences.

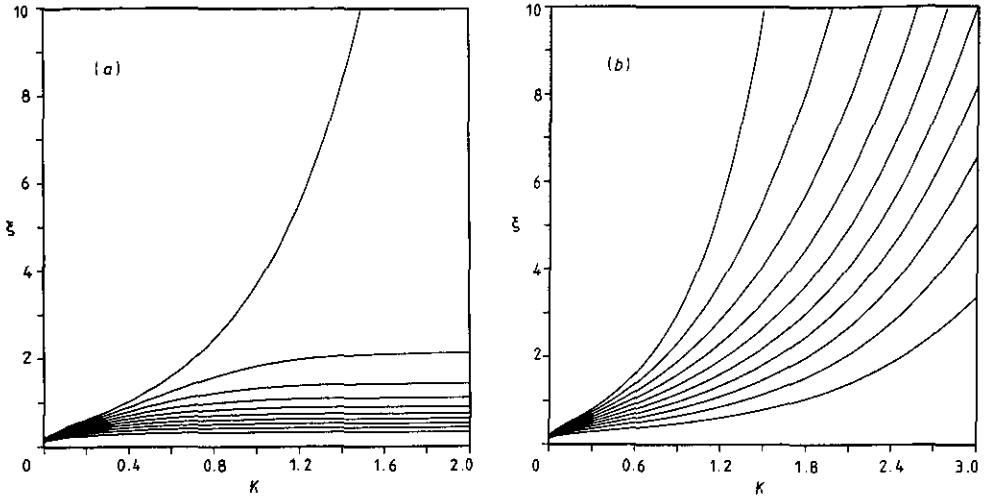


Figure 2. The correlation length, $\xi \equiv -x/\ln(s_0 s_x)$, as a function of βJ for the system of figure 1 with varying q ($q = 0$ to $q = 0.9$, increasing from top to bottom) and different rates: (a) (2.11); (b) (2.13).

3.2. Susceptibility

The susceptibility $\chi = \sum_x \langle s_0 s_x \rangle = (1 + u)/(1 - u)$. Consequently, $\chi \approx 1 + 2(1 - q)K_0 + \dots$ at a high enough temperature. At low temperatures, case (i) is characterized by $u \approx 1 - 2(q/\mu)^{1/2}$ and $\chi \approx (\mu/q)^{1/2}$ as $q \rightarrow 0$. One may compare this with the equilibrium result $\chi \approx 1/q$. Case (ii), however, produces under the same conditions: $u \approx 1 - 2e^{-2\pi K_0}$ and $\chi = \exp(2\pi K_0)$ implying $\gamma = \pi\gamma_{eq} = 2\pi$. This is in agreement with the conclusion in the preceding paragraph.

3.3. Clusters distribution

The cluster distribution is $P_n = 2^{-(n+1)}(1 - u)^2(1 + u)^{n-1}$, $\sum_n P_n = \frac{1}{2}(1 - u)$, and the mean cluster size then follows as $\langle n \rangle = 2/(1 - u)$. Thus, $\langle n \rangle = 2[1 - (1 - q)K_0]^{-1}$ as $K_0 \rightarrow 0$: the clusters tend to become smaller as T and q are increased, as one would expect. When $K_0 \rightarrow \infty$, we get $\langle n \rangle \approx (\mu/q)^{1/2}$ leading to $\gamma = \frac{1}{2}$ for case (i), and $\langle n \rangle \approx \exp(2\pi K_0)$ for case (ii); one has $\langle n \rangle \approx 2\xi$ in both cases.

3.4. Energy and its fluctuations

As mentioned above, our system admits two interpretations. On the one hand, one may imagine that J is changed all over the system at each kinetic step according to $p(J)$. Each configuration may then be characterized by a random energy:

$$E_1(s) = -J \sum_{NN} s_x s_x \tag{3.2}$$

where J represents a random variable sampled independently for each configuration from $p(J)$. Thus, $\sum_s P^{st}(s) E_1(s)$ is the usual average over configurations. This still has a random character, however, and one may thus naturally define a mean energy as

$\int dJ p(J) \langle E_1 \rangle$. Then, according to that interpretation of the changes in J induced by kinetics, averages should be computed in general as

$$\langle A \rangle_1 = \int dJ p(J) \sum_s P^{st}(s) A(s; J). \quad (3.3)$$

On the other hand, Glauber spin-flip processes have a local nature and, consequently, one may assume instead that J is only changed when connecting the spin involved by the attempted transition with one of its NN. Then, the (random) energy which may be associated with each configuration in the stationary state is

$$E_2(s) = - \sum_{NN} J_{(x,x')} s_x s_{x'}. \quad (3.4)$$

Here, $J_{(x,x')}$ is randomly distributed according to $p(J)$ both spatially for each configuration and between successive configurations for each NN pair (x, x') . Thus, the averages are in this case

$$\langle A \rangle_2 = \int dJ P(J) \sum_s P^{st}(s) A(s; J) \quad (3.5)$$

where

$$P(J) = \prod_{(x,x')} p(J_{(x,x')}). \quad (3.6)$$

It follows from (3.2)-(3.6) that the (mean) energy is simply given by

$$\langle E \rangle = - \langle J \rangle \left\langle \sum_x s_x s_{x+1} \right\rangle \quad (3.7)$$

for both interpretations. That is, $e \equiv \langle E \rangle N^{-1} = - \langle J \rangle u = -(1-q)uJ_0$ for the system of interest; the behaviour of u is qualitatively similar to the one for K_c in figure 1.

The energy fluctuations, however, essentially differ for those two versions of the model. It follows from (3.2) and (3.3) and (3.4) and (3.5), respectively, that $\langle E^2 \rangle_1 = \langle J^2 \rangle \langle (\sum_x s_x s_{x+1})^2 \rangle$ and $\langle E^2 \rangle_2 = \langle J^2 \rangle \langle (\sum_x s_x s_{x+1})^2 \rangle + 2N(\langle J^2 \rangle - \langle J \rangle^2)$. Consequently, the square mean fluctuations of the energy, $\Delta E^2 \equiv \langle E^2 \rangle - \langle E \rangle^2$, are

$$\Delta E^2 = \langle J^2 \rangle [\langle u^2 \rangle - \langle u \rangle^2] + (\langle J^2 \rangle - \langle J \rangle^2) \langle u \rangle^2 \quad (3.8)$$

and

$$\Delta E^2 = \langle J \rangle^2 [\langle u^2 \rangle - \langle u \rangle^2] + 2N(\langle J^2 \rangle - \langle J \rangle^2) \quad (3.9)$$

respectively. The last term in (3.9) is of order N implying that the relative magnitude of fluctuations, $[\Delta E^2]^{1/2} / \langle E \rangle$, is of order $N^{-1/2}$. Consequently, those fluctuations vanish within the limit $N \rightarrow \infty$, as for equilibrium. The last term in (3.9), however, is of order N^2 . Consequently, the relative magnitude of fluctuations is then finite in the thermodynamic limit, as one should expect for the first interpretation of the model where kinetics may induce energy changes of order JN at each step. This seems the only essential difference between the macroscopic behaviour of the two interpretations mentioned.

We may also define a non-equilibrium specific heat given by

$$C_v \equiv \partial e / \partial T = -k_B(1-q)K_0^2(1-u^2)[\partial K_c / \partial K_0]. \quad (3.10)$$

This implies a rich critical behaviour. It is also interesting to consider the relation between C_v and ΔE^2 which reveals a general failure of the fluctuation-dissipation theorem. In fact, this is prevented by the last term in equations (3.8) and (3.9), which represent specific non-equilibrium fluctuations, and by the more standard fluctuations represented by the first term of ΔE^2 , which differ from those measured by (3.10).

3.5. Entropy

Finally, one may define an effective entropy as

$$S = -k_B \sum_s P^{st}(s) \ln P^{st}(s) \tag{3.11}$$

cf (2.5). It follows that $Sk_B^{-1} = -K_e \tanh K_e + \ln(2 \cosh K_e)$ and, for the specific distribution $p(J) = (1 - q)\delta(J - J_0) + q\delta(J + J_0)$, for instance, one gets

$$Sk_B^{-1} = \ln 2 - \frac{1}{2}(1 - 2q)^2 K_0^2 + O(K_0^4) \tag{3.12}$$

as $K_0 \rightarrow 0$, and

$$Sk_B^{-1} = \ln(2 \cosh \kappa) + \kappa \exp(-4K_0) \tanh \kappa - [\kappa + \kappa \exp(-4K_0)] \times [\tanh \kappa + \kappa \exp(-4K_0)][1 - \kappa \exp(-4K_0) \tanh \kappa] \tag{3.13}$$

as $K_0 \rightarrow \infty$; here, $\kappa \equiv \frac{1}{4} \ln[(1 - q)q^{-1}]$ and $\kappa \equiv \frac{1}{4}(1 - 2q)[q(1 - q)]^{-1}$.

4. Impure bonds of large strength

In addition to the case of bond dilution, it is interesting to consider the distribution $p(J) = q\delta(J - J_0) + (1 - q)\delta(J + J_1)$ with $J_0, J_1 > 0$, particularly when q is near unity and $J_1 \gg J_0$, say $J_1 \rightarrow \infty$ (see, for instance, Labarta *et al* 1988). The effective coupling constant (2.16) becomes $K_e = -\frac{1}{4} \ln\{[q\phi(4K_0) + (1 - q)\mu]/q\phi(-4K_0)\}$ for that distribution when $J_1 \rightarrow \infty$. For case (i) kinetics, this reduces to $K_e \approx -\frac{1}{4} \ln\{[(1 - q)\mu + q]q^{-1}\}$ when $T \rightarrow \infty$, and to $K_e \approx -\frac{1}{4} \ln[(1 - q)q^{-1}]$ when $T \rightarrow 0$. This means, in particular, that, contrary in a sense to the situation in section 3, kinetics now influences any state, even the infinite temperature state, with the only exception of the ground state (where spins need to point out in one of the two possible directions anyway) which is independent of kinetics. Moreover, the effective interaction K_e is negative when $T \rightarrow \infty$ for any value of q , and it remains so when $T \rightarrow 0$ as far as $q < \frac{1}{2}$, whereas it then changes sign for $q > \frac{1}{2}$. Consequently, as far as $q > \frac{1}{2}$, we may define K_0^c by means of condition $q\phi(4K_0^c) + (1 - q)\mu = q\phi(-4K_0^c)$ which locates the transition between two regions of the phase diagram whose behaviour is predominantly ferromagnetic and antiferromagnetic respectively. For example, $K_0^c = -\frac{1}{4} \ln[(2q - 1)q^{-1}]$ for rates (2.14) and $K_0^c = -\frac{1}{4} \ln(2q - 1)$ for rates (2.11); in both cases, $K_0^c \rightarrow 0$ when $q \rightarrow 1$ and $K_0^c \rightarrow \infty$ when $q \rightarrow \frac{1}{2}$. Note, however, that this transition is smooth; in particular, the correlation length and other quantities remain analytical there.

The situation is less puzzling for case (ii) kinetics, for example one obtains that the effective temperature goes to zero as $J_1 \rightarrow \infty$ for any value of T and q .

5. Comparison with standard models

In principle, our systems may be related to a series of familiar equilibrium disordered systems, namely, Ising models with either quenched or annealed impurities (see, for instance, Stinchcombe 1983). We investigate that possibility in this section.

5.1. *Quenched impure systems*

One may define a quenched or frozen-in impure (equilibrium) Ising model on a 1D lattice (Fan and McCoy 1969) via the canonical partition function:

$$Z_N = \sum_s \exp[-\beta H_N(s)] = \sum_s \prod_x V_{s_x s_{x+1}} \tag{5.1}$$

where

$$H_N(s) = -\sum_x J_x s_x s_{x+1} \equiv -\beta^{-1} \sum_x \ln V_{s_x s_{x+1}} \tag{5.2}$$

note that spatial variations of the bond strength are allowed for here. Then, the transfer matrix method, for example, leads to the free energy per site

$$f = N^{-1} \sum_x \ln[2 \cosh(\beta J_x)] = \langle\langle \ln[2 \cosh(\beta J)] \rangle\rangle \tag{5.3}$$

$x = 1, 2, \dots, N$, and to the energy per site

$$u \equiv N^{-1} \sum_x \langle s_x s_{x+1} \rangle = \langle\langle \tanh(\beta J) \rangle\rangle \tag{5.4}$$

where $\langle\langle \dots \rangle\rangle$ now represents an average with respect to the (normalized) distribution $p(J)$ used to sample the *spatially varying* bond strength. The last terms in equations (5.3) and (5.4) follow after requiring that $N \rightarrow \infty$.

Consider the non-symmetrical bond distribution $p(J) = q\delta(J - J_0) + (1 - q)\delta(J - J_1)$. This leads to

$$f = q \ln(2 \cosh K_0) + (1 - q) \ln(2 \cosh K_1)$$

and

$$u = q \tanh K_0 + (1 - q) \tanh K_1$$

where $K_i = \beta J_i$, $i = 0, 1$. When the system has non-magnetic impure bonds, say when $J_0 \neq 0$ and $J_1 = 0$, which corresponds to the case studied in section 3, the following thermodynamics follow: $f \approx \ln 2 + \frac{1}{2}qK_0^2 + O(L_0^4)$, $u \approx qK_0 + O(K_0^3)$ and $C_v \approx -q\beta K_0$ as $K_0 \rightarrow 0$, and

$$f \approx (1 - q) \ln 2 + qK_0 + O[\exp(-2K_0)] \quad u \approx q[1 - 2 \exp(-2K_0)] + O[\exp(-4K_0)]$$

and $C_v \approx -4q\beta K_0 \exp(-2K_0)$ as $K_0 \rightarrow \infty$. The correlation function is $\langle s_0 s_x \rangle = u^x$ and, consequently, the correlation length behaves $\xi^{-1} \approx \ln q$ as $K_0 \rightarrow \infty$, namely, any impurity washes out the pure zero-temperature critical point, as mentioned above.

This situation bears some similarity only with the corresponding one for the non-equilibrium system in section 3 driven by rates (2.11) or (2.14), i.e. case (i) kinetics. The energy u is the same in both cases when $T \rightarrow \infty$, while one gets $u = q$ and

$$u = \{[(1 - q) + q\mu]^{1/2} - (1 - q)^{1/2}\} \{[(1 - q) + q\mu]^{1/2} + (1 - q)^{1/2}\}^{-1}$$

as $T \rightarrow 0$ for the equilibrium and non-equilibrium systems, respectively, the latter behaving $u \approx 1 - 2(\epsilon/\mu)^{1/2}$ when $q = 1 - \epsilon$ with $\epsilon \rightarrow 0$. Consequently, even though there is some similarity between the two cases, only the behaviour of the non-equilibrium system is crucially determined by kinetics. (In fact, this was expected given our comments on the system interpretations in section 1.) Thus, one might argue that the macroscopic differences should be minimized when considering the annealed impure case where kinetics plays some more fundamental role in determining the system thermodynamics. This is not fully confirmed, however, as shown below.

Consider now the case $p(J) = q\delta(J - J_0) + (1 - q)\delta(J - J_1)$ when $J_1 \rightarrow -\infty$, for instance. One gets $u \approx q \tanh K_0 - (1 - q)$. Thus (as occurs for the non-equilibrium case mentioned in the preceding paragraph) the quantity u when $\beta \rightarrow \infty$ is negative for $q < \frac{1}{2}$, while it becomes positive for $q > \frac{1}{2}$, and one may define a critical temperature corresponding to $u = 0$ when $q > \frac{1}{2}$, namely, $K_0^c = -\frac{1}{2} \ln(2q - 1)$.

5.2. Annealed impure systems

The corresponding annealed case (Thorpe and Beeman 1976) may be defined via the partition function:

$$Z_N = \sum_s \int \prod_{x=1}^N dJ_x p(J_x) \exp(\beta J_x s_x s_{x+1}) = \sum_s \prod_x \langle \langle \exp(\beta J_x s_x s_{x+1}) \rangle \rangle. \tag{5.5}$$

For the bond distribution $p(J) = q\delta(J - J_0) + (1 - q)\delta(J - J_1)$, one has

$$f = \ln[2q \cosh(\beta J_0) + 2(1 - q) \cosh(\beta J_1)]. \tag{5.6}$$

It is then convenient to define K_{eff} and ζ such that

$$K_{\text{eff}} s_x s_{x+1} + \zeta \equiv \ln \langle \langle \exp(\beta J_x s_x s_{x+1}) \rangle \rangle. \tag{5.7}$$

Thus

$$K_{\text{eff}} = \frac{1}{2} \ln[\langle \langle e^K \rangle \rangle \langle \langle e^{-K} \rangle \rangle^{-1}] \tag{5.8a}$$

and

$$\zeta = \ln[\langle \langle e^K \rangle \rangle^{1/2} \langle \langle e^{-K} \rangle \rangle^{1/2}]. \tag{5.8b}$$

That is, the system may be described by an *effective Hamiltonian* defined by (5.7). It then follows that

$$f = \ln[\langle \langle e^K \rangle \rangle + \langle \langle e^{-K} \rangle \rangle] \tag{5.9}$$

and one may define the system energy as $u \equiv \tanh K_{\text{eff}}$.

When one uses $\omega_i(s; x) = \exp(-\frac{1}{2}\Delta H)$ in (1.2), equation (2.16) transforms into $K_e = -\frac{1}{4} \ln[\langle \langle e^{2K} \rangle \rangle \langle \langle e^{-2K} \rangle \rangle^{-1}]$. This means that our system has an effective Hamiltonian which is simply related to (5.8a); namely, $K_e = \frac{1}{2} K_{\text{eff}}(2\beta)$ for rates (2.13) (case (ii) kinetics). The fact that the annealed bond-diluted system is close to that of our model versions is not so surprising after all given that, as was proved before, this particular version is simply characterized by $K_e = K_0$ for a pair bond distribution with $\langle \langle J \rangle \rangle = J_0$, i.e. it essentially reduces then to equilibrium. More interesting is the fact that, even though that similarity is quite far from general, an important conceptual relation between equilibrium and non-equilibrium models exists. In fact, equilibrium, annealed and non-equilibrium dilute Ising systems only differ in practice from each other by some (essential, however) property concerning the (dynamical) nature of the bond disorder $p(J)$ which enters the definition of those systems. The distribution is, respectively, frozen-in, in equilibrium at temperature T with the other degrees of freedom, and changing at random as implied by some sort of local process at infinite temperature. Two comments are in order. On the one hand, only the annealed case, where impurities themselves try to minimize their effect, for example by moving towards the interface, turns out to be physically trivial to the understanding of real disordered systems. On the other hand, one may formulate an interesting situation which generalizes those three cases (Garrido and Marro 1991b).

6. Some exact results for $d \geq 2$

In principle, the formalism in sections 2-4 may be applied to $d > 1$. The general theorems (Garrido and Marro 1989) then imply that, for local transition rates having certain symmetry properties, for example the rates enumerated in section 2, an effective Hamiltonian may exist which needs to have the $\mathbb{N}\mathbb{N}$ Ising structure of the original Hamiltonian (1.4). The theorems also state a necessary and sufficient condition for its existence; this is not satisfied, however, by any version of our model system when $d > 1$. Consequently, the 2D and 3D versions cannot bear the quasicanonical nature which is implied by a global detailed balance condition, as we discussed in section 2. This has two main effects. First our systems will depict in general a more complex non-equilibrium behaviour for $d > 1$ than for $d = 1$. Second, other methods of study need to be used for $d > 1$. We follow here two different approaches to obtain information for the latter cases.

We may obtain upper bounds for the critical temperature of the d -dimensional non-equilibrium impure system from the positivity property of the transition rate. That is, one may always write the rates driving a stochastic process as

$$\omega(s; \mathbf{x}) = \frac{1}{2} \omega(\mathbf{x}) \left[1 - s_x \sum_{\alpha} P_{\alpha}(\mathbf{x}) s_{\alpha} \right]. \tag{6.1}$$

Here, $\omega(\mathbf{x}) > 0$, the sum is over all possible different sets of spins, α , $s_{\alpha} = \prod_{r \in \alpha} s_r$, and $P_{\alpha}(\mathbf{x})$ are real functions. Consequently, once $\omega(s; \mathbf{x})$ is known, one has

$$\omega(\mathbf{x}) = 2^{1-N} \sum_s \omega(s; \mathbf{x}) \tag{6.2}$$

and

$$P_{\alpha}(\mathbf{x}) = - \sum_s s_{\alpha} s_x \omega(s; \mathbf{x}) \left[\sum_s \omega(s; \mathbf{x}) \right]^{-1}. \tag{6.3}$$

It may be shown (Liggett 1985) that, when the minimum possible value of $\omega(s; \mathbf{x})$ is positive, i.e. $\delta \equiv \inf_{\mathbf{x}} \omega(\mathbf{x}) [1 - \sum_{\alpha} |P_{\alpha}(\mathbf{x})|] > 0$, the process is *exponentially ergodic*. This means that, for almost any initial distribution of spins (i.e. any probability measure $\mu \in \Omega$) it is $|(s_{\alpha})_{t,\mu} - \int d\nu s_{\alpha}| \leq 2 e^{-\delta t}$ indicating that the system will relax exponentially rapidly in time towards a unique stationary state (i.e. invariant measure ν).

As an illustration, consider a 1D system with $\omega(s; \mathbf{x}) = q\omega_J(s; \mathbf{x}) + (1-q)\omega_{-J}(s; \mathbf{x})$ and $\omega_J(s; \mathbf{x}) = \phi(\beta \Delta H_J)$ using an obvious notation. It follows from the above theorem that a unique phase exists for any $T > T_0$, where

$$T_0^{-1} = \frac{1}{2} \ln \{ [1 + 2(q^2 - q + \frac{1}{2})^{1/2}] |2q - 1|^{-1} \}$$

for rates (2.13), and $T_0 = 0$ for rates (2.12) and (2.14). Consequently, a 1D system can only undergo a phase transition (at $T_c < T_0$) for rates (2.13). A 2D system with rates (2.14) will always be (exponentially) ergodic in so far as $7 \leq 16q \leq 9$ for any finite temperature (i.e. $T_0 = 0$ in this case), while it may have a phase transition when $q \geq 9/16$ or $q \leq 7/16$ for $T < T_0$, where

$$T_0^{-1} = -\frac{1}{4} \ln \{ \frac{1}{3} [4(4q - 1)^2 - 3(9 - 16q)]^{1/2} - \frac{2}{3}(4q - 1) \}.$$

When $d = 2$ and the rates are (2.13), however, $T_0 \neq 0$ exists which is the solution of $(8q - 7)\kappa^4 + 4\kappa^3 + 6\kappa^2 + 4\kappa + 1 - 8q = 0$, $\kappa \equiv \exp(-2/T_0)$, for any q . When $d = 3$ and the rates are (2.14), $T_0 \neq 0$ exists for any q which is the solution of $(5 - 8q)\kappa^3 + 30\kappa^2 + 15(8q - 3)\kappa - 14(8q - 3) = 0$.

The system in section 3, whose kinetics involve non-magnetic bonds with probability q , may be analysed by the same method. The corresponding result for $d=2$ and Metropolis rates is the following. Define $\Delta \equiv \kappa^2 + 4\kappa + 8q[3(1-q)]^{-1} - 7/3$ with $\kappa \equiv \exp(-4K_0)$. The system is ergodic at any T for $\Delta > 0$ and $1 \geq q > 7/15$, while it may undergo a phase transition for $7/15 > q > 0$ at $T_c < T_0$, where $T_0^{-1} \equiv -\frac{1}{4} \ln\{\frac{1}{3}(19-27q)(1-q)^{-1}\}^{1/2} - 2$. Some bounds for $d=2$ and 3 are represented in figures 3 and 4.

Finally, we note a useful representation of the system dynamics at zero temperature. The case $\omega(s; \mathbf{x}) = q\omega_J(s; \mathbf{x}) + (1-q)\omega_{-J}(s; \mathbf{x})$, $\omega_J(s; \mathbf{x}) = \phi(\beta\Delta H_J) = \min(1, e^{-X})$,

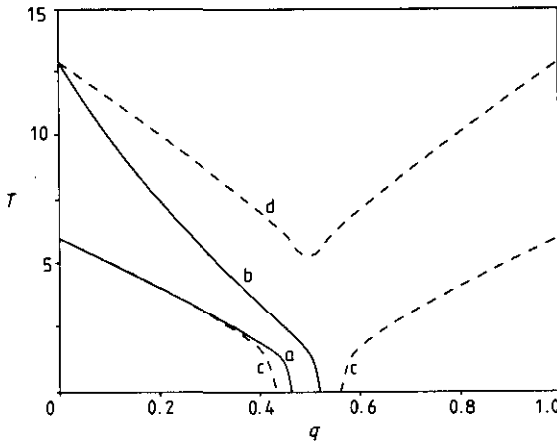


Figure 3. Variation with q of the exact upper bounds in section 6 for T_c in the case of a d -dimensional system evolving via competing spin-flips driven by Metropolis rates: (a) $d=2$ and the bond dilute case $p(J) = (1-q)\delta(J-J_0) + q\delta(J)$, $J_0 > 0$; (b) same $p(J)$ and $d=3$; (c) $d=2$ and the case of a competition between ferro- and antiferromagnetic interactions, $p(J) = (1-q)\delta(J-J_0) + q\delta(J+J_0)$; (d) same $p(J)$ and $d=3$. The system is necessarily ergodic in the upper region defined by the curve at each case.

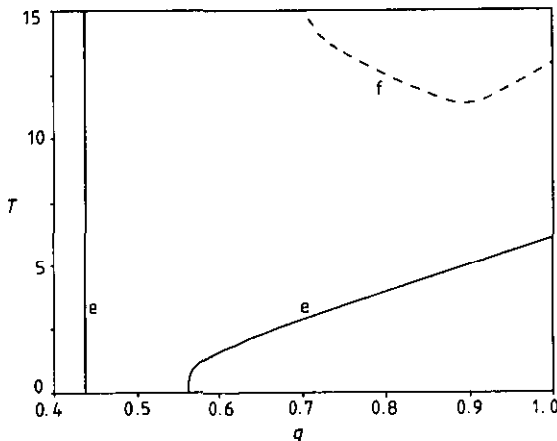


Figure 4. Same as figure 3: (e) $d=2$ and the case of a ferromagnetic system with (antiferromagnetic) impure bonds having very great strength, $p(J) = q\delta(J-J_0) + (1-q)\delta(J+J_1)$, $J_1 \rightarrow \infty$; (f) same $p(J)$ and $d=3$. The latter (broken) line asymptotically tends to $q=0.701$ as $T \rightarrow \infty$.

which we have considered before (see also Garrido and Marro 1991a), is equivalent at $T=0$ to a $\frac{1}{2}$ -spin system on a lattice of arbitrary dimension d which suffers no influence of a heat bath but simply evolves according to the prescription:

$$\omega(s; x) = \begin{cases} q & \text{if } s_x \sum_{|y-x|=1} s_y > 0 \\ 1-q & \text{if } s_x \sum_{|y-x|=1} s_y < 0 \\ 1 & \text{if } s_x \sum_{|y-x|=1} s_y = 0. \end{cases} \tag{6.4}$$

It also follows that the corresponding prescription for the dilute system in section 3 may be represented by

$$\omega(s; x) = \begin{cases} q & \text{if } s_x \sum_{|y-x|=1} s_y > 0 \\ 1 & \text{if } s_x \sum_{|y-x|=1} s_y \leq 0 \end{cases} \tag{6.5}$$

under the same conditions, i.e. $T=0$, Metropolis rates, and arbitrary dimension. (Note that (6.4) and (6.5) are close to the kinetics defining certain familiar cellular automata such as the *voter model* (Liggett 1985).) Now, the theorem above in this section allows us to conclude about the ergodicity of the d -dimensional system at zero T . Namely, the system with $p(J) = (1-q)\delta(J-J_0) + q\delta(J)$, $J_0 > 0$, is ergodic for any q when $d = 1$, for $(7/15) \leq q \leq 1$ when $d = 2$, and for $(35/67) \leq q \leq 1$ when $d = 3$, and the ergodicity of the system with $p(J) = (1-q)\delta(J-J_0) + q\delta(J+J_0)$ is guaranteed for any q when $d = 1$, and for $(7/16) \leq q \leq (9/16)$ when $d = 2$, while our bounds reveal no region when $d = 3$ where the system is necessarily ergodic.

7. Summary of results

A class of non-equilibrium lattice models has been introduced in this paper. Our main motivation was twofold. On one hand, they serve to illustrate the rich variety of non-equilibrium steady states, phase transitions and critical phenomena. On the other, they might contain the required basic ingredients to deepen the present understanding of macroscopic behaviour in some impure systems, namely systems which have either bond dilution or defects consisting of bonds of infinite strength. A remarkable conclusion here is, however, that our latter expectation will only be properly evaluated after solving the models for $d \geq 2$, for example by Monte Carlo or other methods. This would allow a detailed comparison between the behaviour of equilibrium and non-equilibrium models, and a comparison of that behaviour with real experimental data, an objective which is beyond the prospects of the formalism and results in this paper. Here, we report the exact solutions for $d = 1$, and we mainly focus on the properties of the ensuing phase transitions and critical phenomena. We also report exact bounds concerning critical behaviour when $d \geq 2$, and a simple representation of system dynamics at $T=0$ for arbitrary dimension in terms of cellular automata. The latter is relevant for studying the ground state; this is illustrated here by finding ranges of values for the impurity concentration making the system necessarily ergodic at zero T .

The model kinetics consist of a simultaneous superposition of several spin-flip (equivalently, creation-annihilation) mechanisms, as stated in (1.2); each is stochastic and canonical, and acts independently with some probability $p(J)$ as if the involved

coupling constant J has a given (random) value. Under certain conditions, some of our systems are *quasicanonical*, i.e. they may be described sometimes by a NN Ising-like Hamiltonian with an effective coupling constant K_e which depicts in general a rather complex dependence on temperature T . The existence of this quasicanonical property, and the function $K_e = K_e(T)$, are crucially determined by (i) the specific transition rate which controls the elementary kinetic mechanism, (ii) the symmetries of the involved *Hamiltonians*, and (iii) *d*.

The rates which are familiar in the literature for dealing with different problems may be written $\omega_i(s; x) = \phi(\Delta H_i)$. Here, ΔH represents some cost of the transition between successive configurations, and $\phi(X)$ is an arbitrary function with properties $\phi(X) = e^{-X}\phi(-X)$, $\phi(0) = 1$, and $\phi(X) \rightarrow 0$ as $X \rightarrow \infty$. When the system is under zero field, the rates used by Glauber (1963) and Kawasaki (1972) correspond to $\phi(X) = 1 - \tanh(\frac{1}{2}X)$, those used by van Beijeren and Schulman (1984) are for $\phi(X) = e^{-X/2}$, and $\phi(X) = \min(1, e^{-X})$ conforms to the algorithm by Metropolis *et al* (1953). We find that those rates admit two different classes when studying the nature of phase transitions and critical phenomena. Namely, case (i) (e.g. Glauber and Metropolis *et al*) is characterized by the fact that $\phi(-X) \approx \mu > 0$ when $X \rightarrow \infty$; case (ii) (e.g. van Beijeren and Schulman) are such that $\phi(-X) \approx e^{\pi X}$, $\pi < 1$, when $X \rightarrow \infty$.

The non-equilibrium bond-diluted system occurs for $p(J) = (1-q)\delta(J-J_0) + q\delta(J)$. This reveals an interesting behaviour, including the essential dependence of the latter on kinetics. In fact, the pure (equilibrium) zero-temperature critical point washes out for case (i) in the presence of any disorder (i.e. for $q \neq 0$), as is known to occur in the equilibrium 1D system with any non-zero concentration of broken bonds. The zero-temperature critical point remains, however, in case (ii) for any $q \neq 0, 1$, and the associated thermal critical exponents equal π times the equilibrium ones. That is, we find (non-universal) critical behaviour which depends on asymptotic properties of the rate. We also find that the condition $(1-q)\phi(-4K_0) \gg q$ guarantees the existence of a critical point for that bond distribution. Kinetics only become irrelevant at high enough temperature. On the other hand, fluctuation-dissipation relations fail here in general. In fact, the system energy has typically specific non-equilibrium fluctuations, in addition to more standard fluctuations, which however, become, anomalously large in one of our model versions. The system depicts interesting percolation behaviour as well. The mean cluster size $\langle n \rangle$ decreases with increasing T and q , and $\langle n \rangle$ is simply related to the correlation length ξ : $\langle n \rangle = 2\xi$. The latter shows a divergence for case (i) when $q \rightarrow 0$; this is characterized by the critical exponent $\nu = \frac{1}{2}$, independent of kinetics, which is to be compared with the equilibrium value $\nu = 2$. For case (ii), however, one gets $\nu = 2\pi$; thus the system then has a rather complex critical behaviour which markedly differs from the equilibrium case. It is of interest to mention that the quenched dilute spin-glass Ising model has also been reported (Klein *et al* 1989) to have a critical behaviour which differs from that characterizing the standard percolation phenomena in the quenched dilute Ising model.

The non-equilibrium impure system with bonds of infinite strength occurs for $p(J) = q\delta(J-J_0) + (1-q)\delta(J+J_1)$, $J_0, J_1 > 0$, when q is near unity and $J_1 \rightarrow \infty$. In addition to its own interest, the comparison with the case in the previous paragraph illustrates novel features which corroborate the richness of non-equilibrium steady states. In fact, case (ii) now reveals that producing the simplest behaviour, given that the effective temperature then goes to zero as $J_1 \rightarrow \infty$ independently of the values for T and q . In contrast, case (i) produces an essential dependence on kinetics for any temperature except for $T = 0$. Moreover, the phase diagram has two regions which

correspond to predominantly ferromagnetic and antiferromagnetic behaviours, respectively, with a smooth transition located at a temperature which is finite except when $q \rightarrow 1$ or $\frac{1}{2}$.

A main general conclusion is that critical exponents here are typically independent of q and other features, but may depend on asymptotic properties of kinetics such as the shape of $\phi(-X)$ as $X \rightarrow \infty$. This, which adds up to the behaviour shown by other systems with competing kinetics (see, for instance, Garrido and Marro 1991a), indicates that the concept of universality for non-equilibrium systems may be much more subtle than in equilibrium. Using the language of renormalization group techniques, *certain* properties of kinetics may play the role of *relevant* and *marginal* parameters in non-equilibrium theory.

Finally, a detailed comparison between the macroscopic behaviour of our systems and that of familiar equilibrium disordered systems, namely, Ising models with either quenched or annealed impure bonds, is also reported for $d = 1$. Certain occasional similarities exist suggesting, in particular, that (in spite of a general observation at the beginning of this section) our systems may certainly bear some relevance to the understanding of the complex behaviour observed in real materials. We believe that some of the questions raised here, and the systems themselves, deserve more attention; consequently, we are presently studying the case $d \geq 2$ by approximate methods which include Monte Carlo analysis and mean-field theory.

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