Stochastic multi-resonances in complex nets of spiking neurons

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Abstract.   We study the processing of weak signals in a realistic neural medium of spiking neurons by means of the stochastic resonance mechanism. Assuming a Hebbian prescription for the maximal synaptic conductances that balances excitatory and inhibitory synaptic connections, and short-term synaptic plasticity processes affecting such conductances, the system exhibits dynamical well-defined phases. This includes a memory phase, in which population of neurons remain synchronized, an oscillatory phase characterized by jumps between different synchronized populations of neurons, and an asynchronous noisy phase. Increasing noise in the medium when a weak input stimulates each neuron, intriguing phenomena occurs near all the transition points between phases resulting in the emergence of stochastic multi-resonance. This is shown to be quite robust, as it occurs for different levels of synaptic plasticity, type and number of stored patterns, and network topologies. Such robustness suggests that the behaviour shown by the models here should also be observable in actual systems, as a recent psychophysical experiment seems also indicate. Would this be the case, monitoring multi-resonance may help neuroscientists to identify and investigate the existence of phase transitions in the brain, which have been recently associated to some of its high level functions.

1. Introduction

Many physical systems present ambient fluctuations that may play a fundamental role in the functioning of natural systems. They may, for instance, optimize signals propagation by turning the medium into an excitable one, originate order at macroscopic and mesoscopic levels or induce coherence between the intrinsic dynamics of a system and some weak stimuli it receives, a phenomenon known as *stochastic resonance* (SR) (see [1] and references therein). Recently, this last intriguing phenomenon has received the interest of the neuroscience community for its possible implications in the complex processing of information in the brain [2, 3, 4]. In fact, neural systems present the main factors involved in SR, namely, different sources of intrinsic and external noise and complex nonlinear processes affecting excitability of *neurons* and the transmission of their complex signals through the *synapses.*

In the last years, it has been reported that simple *perceptrons* and fully connected networks of binary neurons can efficiently detect weak signals by means of the SR mechanism through more than one level of intrinsic noise [3, 4]. These new stochastic multiresonance phenomena (SMR) are induced by biophysical processes producing short-term synaptic plasticity at the synapses and by the existence of adaptive processes affecting neural excitability. Here we report that this intriguing SMR phenomenon is more general and can also appear in more realistic complex networks of spiking neurons. Moreover, we hypothesize that SMR appears as a consequence of destabilization of dynamical attractors since SR peaks appear around the transitions points between different phases in the system.

2. Models and Methods

Our main findings here has been obtained in a complex network of N spiking neurons. For simplicity, we considered a simple *integrate and fire* description of the neuron activity defined by the following dynamics for the membrane potential V_i of neuron i :

$$\tau_m \frac{dV_i(t)}{dt} = -V_i(t) + RI_i(t) \qquad \text{if } V_i(t) < V_{th}$$
$$V_i(t) = 0 \quad (t_{sp} < t < t_{sp} + \tau_{ref}) \quad \text{if } V_i(t_{sp}) > V_{th}, \tag{1}$$

where τ_m is the cell membrane time constant, R is the membrane resistance, V_{th} is the voltage threshold for neuron firing, t_{sp} is the time at which $V(t) \geq V_{th}$, τ_{ref} is the so called *refractory period* and I_i is the total input current to

neuron i which we assumed to be:

$$I_i(t) = I_i^{ext}(t) + I_i^{syn}(t) + D\zeta(t). \tag{2}$$

Here, $I_i^{ext}(t) = I_0 + I^{signal}(t)$ is the current generated by an external input constituted by a constant term I_0 plus a weak sinusoidal signal $I^{signal}(t) = \delta I \sin(2\pi f_s t)$ with amplitude $\delta I \ll 1$ and frequency f_s. On the other hand, $D\zeta(t)$ is a noisy current with $\zeta(t)$ being a white noise with $\langle \zeta(t) \rangle = 0$ and $\langle \zeta(t)\zeta(t') \rangle = \delta(t - t')$. Finally, $I_i^{syn}(t)$ is the postsynaptic current generated at neuron i due to the activity of its neighbors.

We considered also short-term synaptic plasticity at the synapses using the synapse model introduced in [5]. Here, the state of a given synapse j is described by variables $y_j(t)$, $z_j(t)$ and $x_j(t)$ representing, respectively, the fraction of *neurotransmitters* in active, inactive and recovering states (see [5] for details). Within this framework active neurotransmitters $y_j(t)$ are the responsible for the generation of the postsynaptic response to incoming presynaptic inputs and become inactive after a typical time $\tau_{in} \sim 2 - 3ms$. Thus, the synaptic current is assumed to be $I_i^{syn}(t) = \sum_j A\omega_{ij}e_{ij}y_j(t)$ where e_{ij} is the adjacency matrix, and $A\omega_{ij}$ is the maximal synaptic current that can be generated by a single presynaptic input, normally associated to maximal synaptic conductances, where ω_{ij} are Hebbian synaptic weights. On the other hand, inactive neurotransmitters recover during a typical time τ_{rec} which is order of a half second for typical pyramidal cells [5]. Recovered neurotransmitters become immediately active with some probability U (the so called release probability) every time a presynaptic input arrives to the synapses.

One can associate a binary variable, $s_i(t)$ to each neuron in the network with $s_i(t) = 0$ if $V_i(t) < V_{th}$ and $s_i(t) = 1$ at $t = t_{sp}$, that is when an action potential or spike is generated. In this way, $m(t) \equiv \sum_i s_i(t)$ monitors the mean firing rate, or mean activity, in the network. One can measure the degree of correlation between the network activity and the weak input signal $I^{signal}(t)$ for different level of noise D in the network by computing the function

$$C(D) \equiv \frac{|C_{f_s}|^2}{(\delta I)^2},$$

where $C_{f_s} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t_0}^{t_0+\tau} m(t)e^{if_s t}$ is the Fourier coefficient of the mean firing rate at the weak signal frequency. This function help us to explore the existence of stochastic resonance phenomena in our system for a wide range of neuron, synapse and network topology parameters.
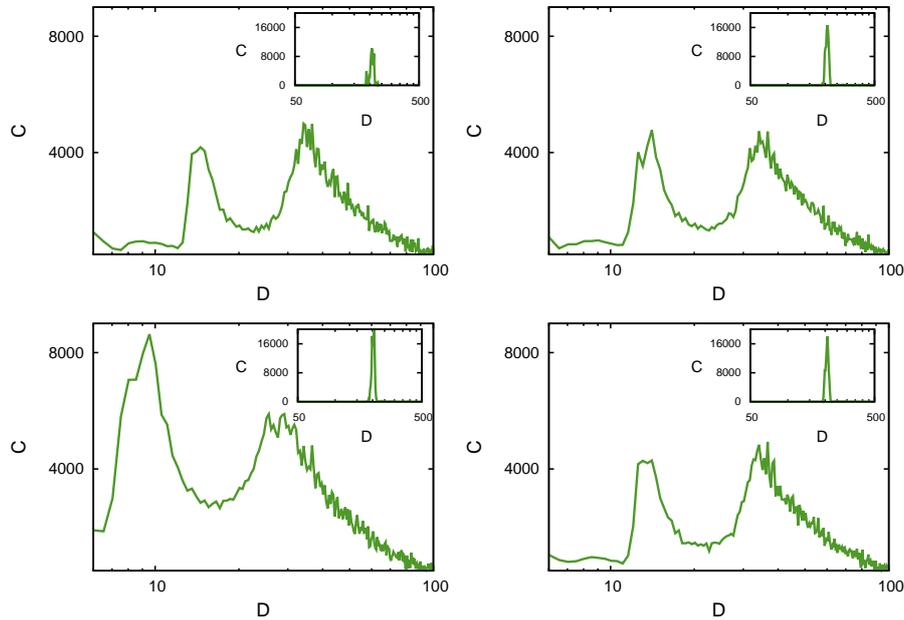
Figure 1: Emergence of SMRs in complex networks: Panels A, B, C and D corresponds, respectively, to a fully connected network, a random diluted network, a scale-free network with $p(k) \sim k^{-3}$ and a Watts-Strogatz small-world network with $p_r = 0.5$. Resonance curves have been obtained for a network of $N = 400$ nodes with a random stored pattern at the synapses after averaging over ten different networks. In cases B, C and D the network mean connectivity was $\langle k \rangle = 100$. In all panels the insets depict single resonance peaks that emerge in absence of short-term synaptic plasticity ($\tau_{rec} = 0$.)

3. Results and conclusions

Figure 1 summarizes our main findings here, that is, the emergence of robust SMRs characterized by the existence of two optimal levels of ambient noise at which network activity strongly correlates with the weak input signal. An extensive analysis of this SMR phenomenon shows that both resonance peaks appear around levels of noise at which phase transitions occur [6]. In particular, the low noise resonance appears when stable memory attractors become metastable originating a new non-equilibrium dynamical phase characterized by jumps of the network activity among these metastable states. The resonance peak then occurs when the typical frequency for jumps is coupled with the frequency of the weak signal. On the other hand, the second resonance

peak at large noise appears when the amplitude of the jumps decreases and their frequency increases consequence of the larger noise and, consequently, a second order phase transition towards a non-memory phase takes place. Around this transition point the potential barrier among attractors is so small that the weak signal can drive the jumps. The consequence is the appearance of a modulation of the noisy activity with the same frequency of the weak signal which results in the emergence of the second resonance peak. We concluded then that the existence of the nonequilibrium dynamical phase between the memory and non-memory phase is a necessary condition for SMR. This phase emerges due to the existence of short-term synaptic plasticity at the synapses. For static synapses this phase is not present and, therefore, only an one-peak SR phenomenon appears around the critical point between the memory and non-memory phases, as it is depicted in the inset graphs o all cases shown in figure 1. The figure also illustrates the robustness of the SMR phenomena for different network topologies including a fully connected network (panel A), random diluted networks (panel B), scale-free networks (panel C) and Watts-Strogatz small-world networks (panel D). Such robustness suggests that SMRs should also occur in actual systems as a recent experimental data suggest [2]. Would this be the case, monitoring SMRs may help neuroscientists to identify and investigate the existence of phase transitions in the brain.

Acknowledgments

References

[1] L. Gammaitoni, P. Hanggi, P. Jung, and F. Marchesoni, *Rev. Mod. Phys.* **70**(1), 223287 (1998).

[2] H. Yasuda, T. Miyaoka, J. Horiguchi, A. Yasuda, P. Hanggi, and Y. Yamamoto, *Phys. Rev. Lett.* **100**(11), 118103 (2008).

[3] J. F. Mejias and J. J. Torres, *PLoS ONE* **6**(3), e17255 (2011).

[4] G. Pinamonti, J. Marro and J. J. Torres, *PLoS ONE* **7**(12), e51170 (2012).

[5] M. V. Tsodyks and H. Markram, *PNAS* **94**(2), 719723 (1997).

[6] J. J. Torres, I. Elices and J. Marro, *to be published* (2014).