Chaos in Heterogeneous Networks with Temporaly Inert Nodes

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Abstract

We discuss an attractor neural network in which only a fraction ρ of nodes is simultaneously updated. In addition, the network has a heterogenous distribution of connection weights and, depending on the current degree of order, connections are changed at random by a factor Φ on short–time scales. The resulting dynamic attractors may become unstable in a certain range of Φ thus ensuing chaotic itineracy which highly depends on ρ. For intermediate values of ρ, we observe that the number of attractors visited increases with ρ, and that the trajectory may change from regular to chaotic and vice versa as ρ is modified. Statistical analysis of time series shows a power–law spectra under conditions in which the attractors' space is most efficiently explored.

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1 Introduction

There has been a great interest in the study of complex networks in physics [Barabasi, 2002; Boccaletti *et al.*, 2006] mainly focusing on the *wiring* topology of the network. Natural and man–made networks exhibit a number of relevant qualities besides interesting topological structure [Boccaletti *et al.*, 2006; Barrat *et al.*, 2004; Serrano *et al.*, 2006; Zhou *et al.*, 2006], however. We are concerned with two features which could affect the a network performance. We consider here networks with *weighted and time–dependent connections* as in trophic webs and social and communication networks. In these examples, in addition to long–time scale variations, weights often change on a short–time scale to improve actual functioning. On the other hand, we consider *partial activation of nodes* which can also induce *fast fluctuations* in the network. One may argue that maintaining all nodes synchronized and fully informed of the activity of *all* the others requires a lot of energy. Moreover, there are indications that certain nodes are more active than others, and that only a fraction is actually engaged at each time in some cooperative tasks. For example, this is a characteristic of excitable media [Cartwright, 2000]. The possibility of having temporarily inert or reticent nodes is also a recent concern in computer science in relation with parallelism [Korniss *et al.*, 2003; Tosic & Agha, 2005], in mathematical–physics [Evans, 1997], and in neuroscience [LeBeau *et al.*, 2005; Wagenaar *et al.*, 2006; Azouz & Gray, 2000; Olshausen & Field, 2004; Shoham *et al.*, 2006].

Time-dependent connections in physics have only recently been investigated [Barrat *et al.*, 2004; Serrano *et al.*, 2006; Zhou *et al.*, 2006]. However, studying the consequences of fast connection changes in biologically inspired models has a two–decades history [Cortes *et al.*, 2006]. In particular, it has recently been shown that the susceptibility of a network to outside influence increases dramatically for excitable nodes [Kinouchi & Copelli, 2006] and, more specifically, under a competition of processes which tend to increase and decrease, respectively, the efficiency of synaptic connections at short times [Torres *et al.*, 2007]. On the other hand, investigating the effect of inert nodes is rare [Tosic & Agha, 2005; Evans, 1997; Herz & Marcus, 1993; Park *et al.*, 2004]. In this paper we report on the combined effect of these two features in an attractor neural network. We show that varying the fraction of nodes that are simultaneously active induces a variety of qualitatively different behaviours in situations of great susceptibility, but not in more general conditions. The susceptibility needed to observe the most interesting behaviour is shown to occur under appropriate tuning of the connection weights with the network activity. As an application of this finding, we describe here unstable dynamics which leads to itinerancy and chaotic behaviour.

2 The model

We consider a network with N nodes, and $s \equiv \{s_i = \pm 1\}$ and $\mathbf{w} \equiv \{w_{ij} \in \mathbb{R}\}$ $(i, j = 1, \ldots, N)$ will represent, respectively, the node states or binary *activities* and the connection weights. From these we define a local field on each node due to the weighted action of the others, namely, $h_i(\mathbf{s}, \mathbf{w}) \equiv \sum_{j \neq i} w_{ij} s_j$. At each time unit, the activity of n nodes is updated according to a probabilistic master equation with a transition probability $\mathbf{T}(\mathbf{s} \to \mathbf{s}') = \sum_{\mathbf{x}} p_n(\mathbf{x}) \prod_{\{i|x_i=1\}} \tau_n(s_i \to s_i') \prod_{\{i|x_i=0\}} \delta_{s_i, s_i'}$. Here, $\mathbf{x}$ is an operational set of binary indexes —fixed to 1 at n sites chosen at each time according to distribution $p_n(\mathbf{x}) = \binom{N}{n}^{-1} \delta\left(\sum_i x_i - n\right)$, and fixed to zero at the other $N - n$ sites. This choice simply states that one (only) updates simultaneously the selected n nodes. The corresponding elementary rate is $\tau_n(s_i \to s_i') = \sigma(s_i \to s_i') \left[1 + \left(\delta_{s_i', -s_i} - 1\right) \delta_{n,1}\right]$, where $\sigma = \sigma(\mathbf{s}, \beta)$ is a function of (only) $\beta s_i h_i$, with β an inverse temperature parameter. The above describes *parallel updating*, as in cellular automata, for $n = N$ or, macroscopically, $\rho \equiv n/N \to 1$. However, the model describes *sequential updating*, as in kinetic magnetic models, for $n = 1$ or $\rho \to 0$. We are interested in changes with $\rho \in (0, 1)$. This amounts a sensible generalization of familiar cellular automata and represents some real situations, as indicated in the introduction. For example, assuming a neural network, ρ may stand for the fraction of neurons that are stimulated each cycle; the rest maintain the information from the previous state.

This model may be simulated in a computer for different transition details which imply evolution towards non-equilibrium steady states, which is known to be realistic [Marro & Dickman, 1999]. We shall assume that fields are $h(\mathbf{s}, \mathbf{w}) = h[\pi(\mathbf{s}), \xi_i]$. Here, $\xi_i \equiv \{\xi_i^\mu = \pm 1; \mu = 1, \ldots, M\}$ stands for M given realizations of the set of activities, or *patterns,* and $\pi \equiv \{\pi^\mu(\mathbf{s})\}$, where $\pi^\mu(\mathbf{s}) = N^{-1} \sum_i \xi_i^\mu s_i$ measures the *overlap* between the current state and pattern μ. For $N \to \infty$ and finite M, i.e., in the limit $\alpha \equiv M/N \to 0$, the time equation $\pi_{t+1}^\mu(\mathbf{s}) = \rho N^{-1} \sum_i \xi_i^\mu \tanh\{\beta h_i[\pi_t(\mathbf{s}), \xi_i]\} + (1 - \rho)\, \pi_t^\mu(\mathbf{s})$ follows for any μ.

The above allows to monitor the resulting behaviour for different relations between the fields h_i and other network properties. The simplest case of a relation of this kind is Hopfield's [Hopfield, 1982] which follows here for $\rho \to 0$ and weights fixed according to the Hebb prescription, i.e., $w_{ij} = N^{-1} \sum_\mu \xi_i^\mu \xi_j^\mu$. The symmetry $w_{ij} = w_{ji}$ then assures $P_{t \to \infty}(\mathbf{s}) \propto \exp\left(\beta \sum_i h_i s_i\right)$. This (equilibrium) case exhibits *associative memory* prop-

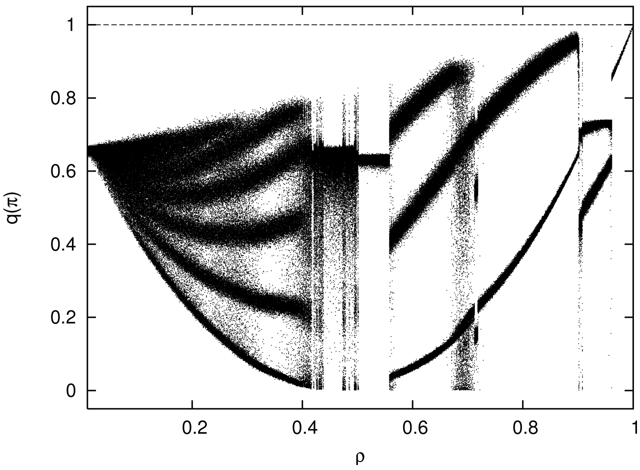

Figure 1: Evidence of chaos. Bifurcation diagram showing the stationary order parameter $q\left(\pi\right)$, as defined in the main text, *versus the synchronization* parameter ρ for $M = 5$ random patterns, $N = 1600$ nodes, $\beta = 100$ and $\Phi = -1/2$. This behaviour is characteristic of any $\Phi \neq 1$, and it follows indistinctly from the analytical solution and from Monte Carlo simulations. The dashed line on top corresponds to the Hopfield equilibrium case.

erty. That is, for high enough β, the patterns $\{\xi_i\}$ are attractors of dynamics [Amit, 1989], as if they would have been *stored* in the connections and recalled in the course of the system relaxation with time.

Equilibrium is generally impeded for $\rho > 0$ [Grinstein *et al.*, 1985], and the asymptotic state then strongly depends on dynamic details [Marro & Dickman, 1999; Ódor, 2004]. We checked that, in agreement with some indications [Herz & Marcus, 1993], the Hopfield–Hebb network also exhibits associative memory for $\rho > 0$. However, no new physics emerges as ρ is varied in this case, and it is likely this occurs rather generally concerning dynamics for simple weighted networks. On the contrary our model exhibits rather complex behaviour for varying ρ if one assumes activity dependent weights. This is expected to occur in many excitable media [Cartwright, 2000], and it has been well documented in the literature concerning the brain. In this case, transmission of information and computations have repeatedly been reported to be correlated with activity–induced fast fluctuations of synapses, i.e., our w_{ij}'s [Ferster, 1996; Abbott & Regehr, 2004]. For example, it has

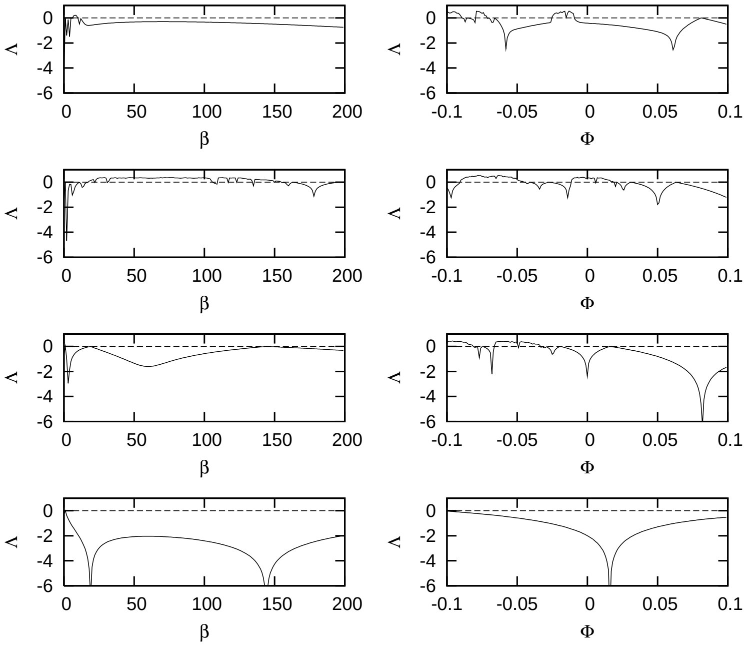

Figure 2: The Lyapunov exponent Λ as a function of the β and Φ for several values of $\rho = 0.25, 0.5, 0.8, 0.99$, from bottom to top, respectively. $\Lambda > 0$ correspond to an irregular or chaotic dynamics. Left graphs correspond to $\Phi = 0.01$ and Λas a function of β,and right panels correspond to $\beta = 25$ and Λas a function ofΦ. Note that there is a small chaotic region ($\Lambda > 0$) for $\rho \lesssim 1$ and low β and $\Phi = 0.01$.

been observed that the efficacy of synaptic transmission can undergo short–time increasing (sometimes called *facilitation*) [Reyes *et al.*, 1998; Wang *et al.*, 2006] or decreasing (*depression*) [Thomson & Deuchars, 1994; Abbott *et al.*, 1997], and that these effects depend on the activity of the presynaptic neuron. It has already been demonstrated that such processes may importantly affect a network performance [Cortes *et al.*, 2006; Torres *et al.*, 2007; Bibitchkov *et al.*, 2002; Pantic *et al.*, 2002; Romani *et al.*, 2006]. Likewise, it seems sensible to assume that similar short–time variations may occur in other networks —e.g., reaction–diffusion systems and the cardiac tissue [Cartwright, 2000]— associated with some efficacy lost after heavy work or with excitations, for instance.

We therefore shall assume that the connection weights are $w_{ij} = \varepsilon_{ij}\overline{w}_{ij} = \varepsilon_j \overline{w}_{ij}$, where the second equality is introduced for simplicity. Here, $\overline{w}_{ij}$ stands for some reference value and ε_j for a random variable. That is, we are assuming some "noise" on top of a previous preparation of the connections designed so that the network can perform some specific function. This also suggests us to assume that the random variable ϵ_j is fluctuating very rapidly so that, on the time scale for the activity changes, it behaves as stationary with distribution given, for example, by $p^{\mathrm{st}}(\mathbf{s}, \varepsilon_j) = q\delta(\varepsilon_j - \Phi) + (1-q)\,\delta(\varepsilon_j - 1)$. We shall further assume that q depends on the degree of *order* in the system at time t, namely, that $q = q(\pi_t)$. For the sake of concreteness, our choices here will be that $q(\pi) = (1+\alpha)^{-1} \sum_\mu \pi^\mu(\mathbf{s})^2$ and that $\overline{w}_{ij}$ is given by the Hebb prescription. The result is that each node is acted on by an effective field $h_i^{\mathrm{eff}}(\mathbf{s}, \mathbf{w}) = \sum_{j \neq i} w_{ij}^{\mathrm{eff}} s_j$ with

$$w_{ij}^{\mathrm{eff}} = [1 - (1-\Phi)\, q(\pi)]\, \overline{w}_{ij}. \tag{1}$$

This amounts to assume short–term variations affecting the intensity of connections by a factor, either positive or negative, Φ on the average. More specifically, one has a decreasing effect for any $\Phi < 1$, and enhancement for $\Phi > 1$, as far as $\Phi > 0$, while $\Phi < 0$ allows for the possibility of a change in the nature of the weights. For the indicated choices of fields and reference weights, our framework reduces to the familiar Hopfield–Hebb case for $\Phi = 1$. It should not be difficult to implement the model for different choices of both $p^{\mathrm{st}}(\mathbf{s}, \varepsilon_j)$ and weights w_{ij}.

3 Results

Assuming the effective weights (1), it readily ensues, from the dynamics of π_t^μ in the case of $M = 1$, that $\pi_\infty = F(\pi_\infty; \rho, \Phi)$. Local stability requires that $|\partial F / \partial \pi| < 1$, where $F(\pi; \rho, \Phi) \equiv \rho \tanh\{\beta\pi[1 - (1-\Phi)\pi^2]\} + (1-\rho)\pi$. Therefore, fixed points are independent of ρ for any Φ, but stability demands that $\rho < \rho_c$ with $\rho_c = 2\left\{3\beta\pi_\infty^2\left[\left(\frac{4}{3}-\Phi\right) - (1-\Phi)\pi_\infty^2\right] - \beta + 1\right\}^{-1}$. The resulting situation for any $\Phi \neq 1$ is illustrated in Fig. 1, where one observes regular behaviour, bifurcations and chaotic windows. This picture cannot occur for fixed weights, e.g., in the Hopfield case (dashed line). In order to deepen on the possibility of chaos, we computed the Lyapunov exponents for different values of ρ as a function of the relevant parameters, namely Φ and β, from the analytical solution for $M = 1$. This is shown in

Fig. 2. The figure clearly reveals the existence of chaos above some degree of synchronization, more specifically, for $\rho > \rho_c\,(\beta, \Phi)$ which marks the onset of period doubling before irregular behaviour. For example, the left graphs show that, for a small positive value of Φ, which corresponds to some slight depression of connections which occurs more likely the higher the current system order is, there is a region for large β (relatively small temperature, say $T \approx 0.02$ in our arbitrary units) and $1 > \rho \gtrsim 0.8$ for which dynamics may eventually become chaotic. In the same graph one may notice a tiny chaotic window for $\rho \approx 1$ and $\beta \approx 7$; this is the case identified previously by us [Marro *et al.*, 2007]. The right graphs, on the other hand, illustrate that chaos is typically an exception for positive values of Φ; it may only occur then for a rather large fraction of synchronized nodes (large ρ) near $\Phi \lesssim 0$. On the contrary, for negative Φ, i.e., when the order tends to induce changes in the nature of the connection intensities, it is more likely that the system will behave chaotically. It is also to be remarked that, inside the chaotic region in each graph, there is a complex pattern of transitions from regular to irregular behaviour as one changes, even very slightly the values of ρ, Φ and β. The next question is whether such complex behaviour may have some constructive role in natural and man–made networks.

The behaviour the system exhibits is illustrated with some *stationary* Monte Carlo runs in Fig. 3. One observes: (a) for $\rho < \rho_c$, convergence towards one of the attractors, namely, fixed points corresponding to the patterns provided, which is revealed by the fact that one of the overlaps (the red one) is large, close to 1, while the others two are closer to zero; (b) irregular behaviour with positive Lyapunov exponent for a larger value of ρ where that dynamics is now unstable and the system activity is visiting the different attractors, including the negative of some of them or *antipatterns;* (c) a different type of irregular behaviour in which, in addition to visiting different attractors on a large time scale, there are much more rapid irregular transitions between one pattern and its antipattern; and finally (d) regular oscillation between one attractor and its negative, which is more rapid as $\rho \to 1$ (that is, when all the nodes are active). The cases (b) and (c) are examples of instability–induced switching phenomena, namely, the system describes in these cases kind of heteroclinic paths among the attractors, and remains different time intervals in the neighbourhood of each of them, as it was previously observed in a related case [Marro *et al.*, 2007].

The constant switching of the activity from one pattern to the other seems to be a rather robust phenomenology. In particular, it also occurs chaotic

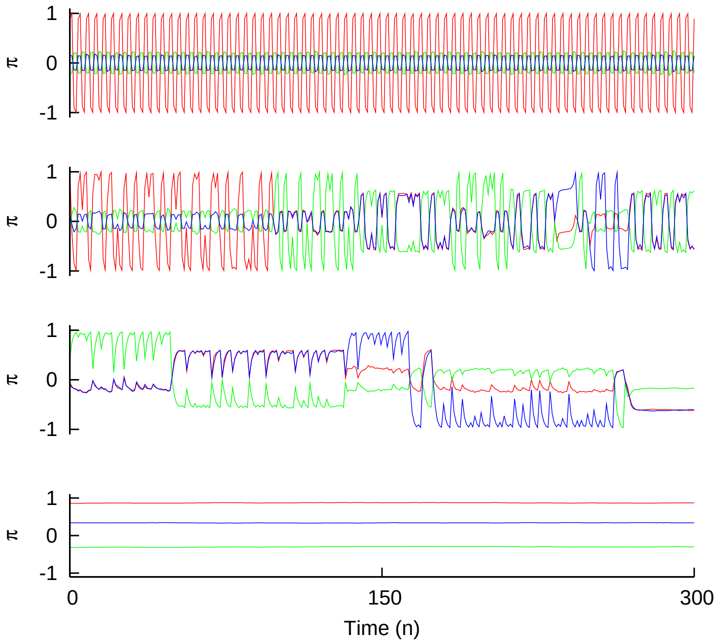

Figure 3: Typical Monte Carlo runs. This shows the overlap as a function of time (in units of n MC trials), during the stationary regime after equilibration, for $N = 1600$ nodes, $\beta = 50$, $\Phi = 0.0035$ and, from bottom to top, $\rho = 0$, 0.59, 0.89, and 0.95, respectively. In this case, $\rho_c \simeq 0.5$. This is for $M = 3$ correlated patterns (identified here with different colours). That is, we generated three patterns completely at random, and then replaced 20% of the digits in the second and third patterns with the same number of digits, and flipped digits, respectively, taken from the first pattern.

hoping for an infinite system and rather arbitrary sets of patterns. In order to illustrate this fact, consider patterns for which $p\left(\xi_i^\mu\right) = \frac{1}{2}\left(1+a\right)\,\delta\left(\xi_i^\mu - 1\right) + \frac{1}{2}\left(1-a\right)\,\delta\left(\xi_i^\mu + 1\right)$, so that $\langle \xi_i^\mu \rangle = a$. Assuming only two patterns, the dynamics for π_t^μ $(\mu = 1, 2)$ may be worked out explicitly, namely, one has for the overlaps that

$$\pi_{t+1}^{(i)} = \frac{1}{2}\rho\left(1+a^2\right)\tanh A_+\left(\pi_t\right) + \frac{1}{2}\rho\left(1-a^2\right)\tanh A_-\left(\pi_t\right) + \left(1-\rho\right)\pi_{t+1}^{(i)}, \tag{2}$$

$i = 1, 2$, where $A_\pm\left(\pi\right) \equiv \beta\left[1-\left(1-\Phi\right)q\left(\pi\right)\right]\left(\pi^{(1)} \pm \pi^{(2)}\right)$. The numerical analysis of these two coupled mean–field equations reveals switching qualitatively similar to the one observed in the simulations for appropriate values of ρ, β, Φ and a, as shown in figure 4. The parameter region for which chaotic switching among the two patterns occurs tends to decrease as $a \longrightarrow 0$, which generally corresponds to fully orthogonal patterns, a case with little interest in practice. The phase space graphs at the right confirm the same.

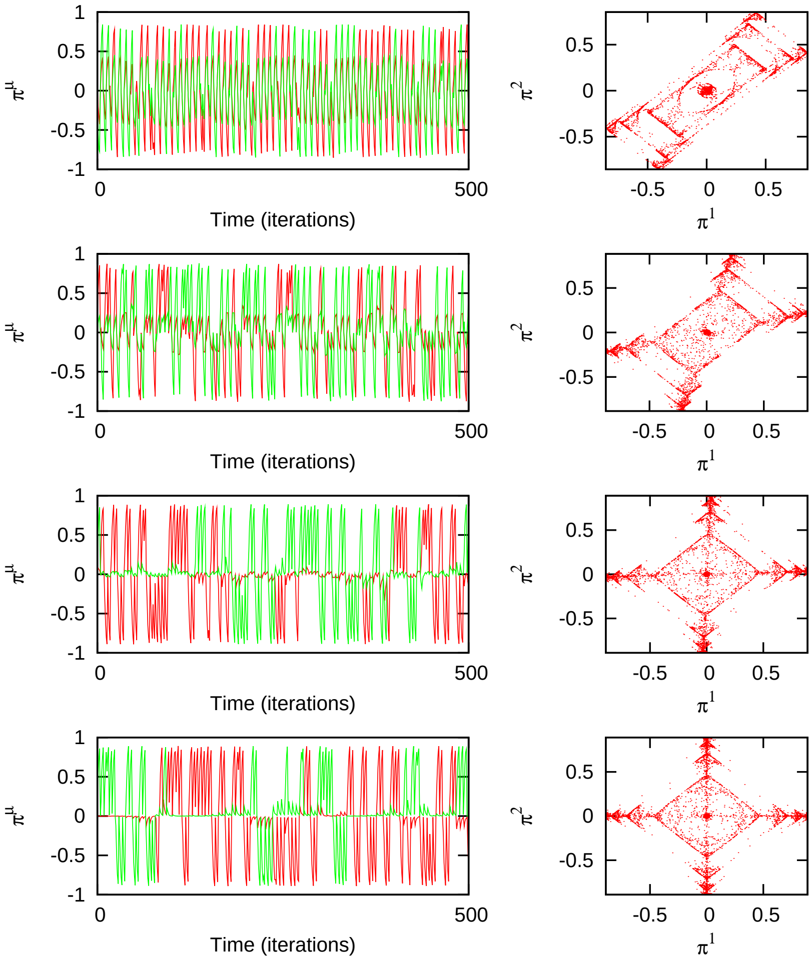

Figure 4: (Left) Time evolution showing the nature of the hoping among the two random patterns and their antipatterns, as obtained from equations 2 for $\Phi = -\frac{1}{2}$, $\beta = 100$, $\rho = 0.46$ and, from top to bottom, $a = 0.7$, 0.5, 0.2 and 0, respectively. (Right) Phase portraits showing $\pi^{(2)}$ versus $\pi^{(1)}$ to deepen on the same behaviour.

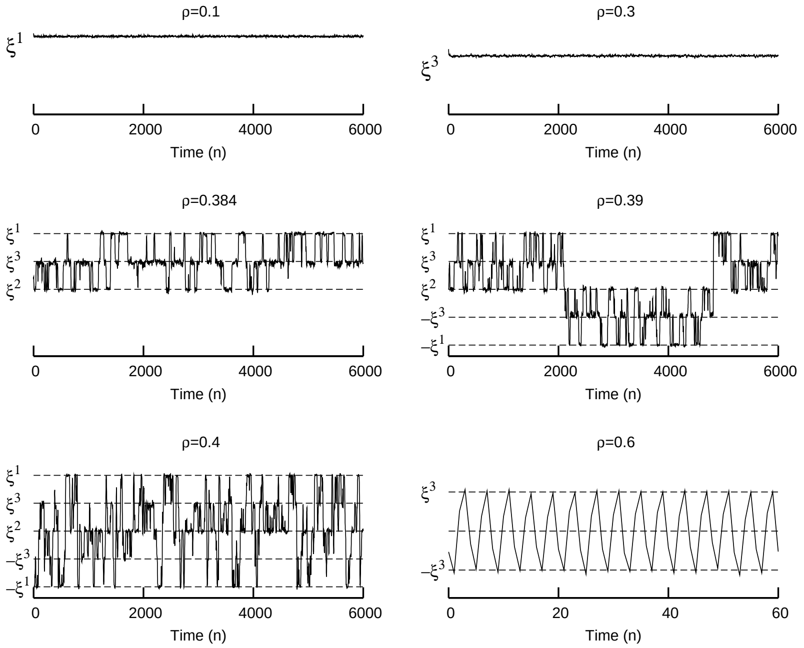

Figure 5: Mote Carlo runs that one may interpret as *states of attention* in the network, which illustrates the possible role of chaos. This shows time-series of the mean firing rate for $N = 1600$, $\beta = 167$, $\Phi = -\frac{1}{2}$, and different values of the paramenter $\rho = 0.1$,0.3, 0.384, 0.39 and 0.4. Here, $\rho_c = 0.38$, and the system stores three patterns, ξ^μ, $\mu = 1, 2$ and 3, as described in the main text. The figure shows how the number of visited attractors increases with ρ after ρ_c. Note that although there three stored patterns ξ^μ, the antipatterns $-\xi^\mu$ are also attractors of the networks dynamics.

An interesting fact concerning the nature of temporal itineracy among the stored patters as ρ is varied is illustrated in Fig. 5. This shows time evolution of the mean firing rate defined as

$$m = \frac{1}{2N} \sum_{i=1}^{N} (1 + s_i)\,. \tag{3}$$

Three patterns (and their corresponding antipatterns) are involved here which consist of a string of 1s, a string with the first 50% positions set to 1 and the rest to -1, and a string with only the first 20% positions set to 1, respectively.

In this Monte Carlo experiment, the activity remains wandering around one of the patterns for any $\rho < \rho_c$. The choice of pattern depends on the initial condition (top graphs in the Fig. 5). For larger values of ρ within a chaotic window (middle graphs and left-bottom graph in Fig. 5), the system tends to visit the other patterns as well. In particular, the middle-left graph in the figure ($\rho = 0.384$) shows visits to the three patterns, and a trajectory which is structured, namely, there are many jumps between the pairs of more correlated patterns, and only a few between the most distant ones. Moreover, the number of jumps between the less correlated patterns tends to grow as ρ is further increased within the chaotic window. The figure shows that, for $\rho = 0.39$ and 0.40, even the antipatterns are visited; note that we have that $\xi^2 = -\xi^2$. Increasing ρ further, e.g., for $\rho = 0.6$ in this specific experiment, the network surpasses equiprobability of patterns and, eventually, abandons the chaotic regime to fall into a limit cycle (bottom-right graph), where it periodically oscillates between a pattern and its antipattern.

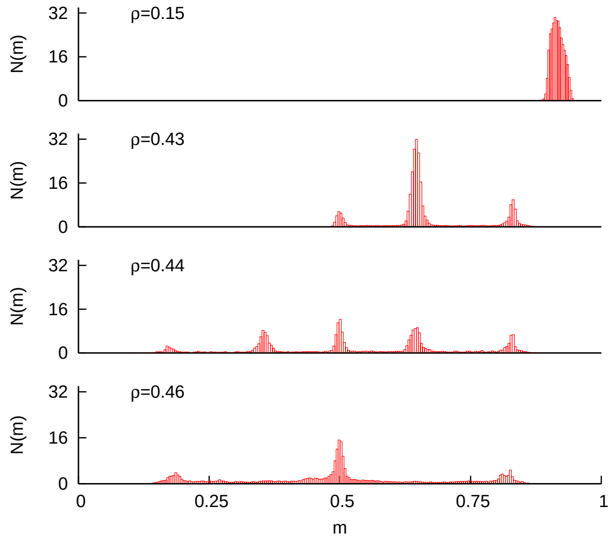

Figure 6: Monte Carlo histograms showing qualitative changes in the distribution of the mean firing rate m, which is a measure of the time the system is around a particular pattern or antipattern. This is for time series with patterns defined as in Fig. 5, $N = 1600$, $\beta = 167$ and $\Phi = 0.05$, and it follows that now $\rho_c \simeq 0.43$.

This interesting behaviour is made more explicit in Fig. 6, where we plotted the distribution probability for the mean firing rate, $\mathcal{P}(m, \rho) = kN(m)$, with k a normalization constant and $N(m)$ the number of occurrences that m has a value between m and $m + dm$ during a large temporal window Δt. This tell us how often the activity of the network is around a particular memory pattern, and how this is affected when one varies ρ. The figure show that for ρ small $\mathcal{P}(m, \rho)$ is centered around the mean activity of one of the stored patterns. As ρ increases the variance of the distribution also increases, and for $\rho > \rho_c$ $\mathcal{P}(m, \rho)$ becomes multimodal with several peaks centered around the mean-activity associated to all stored patters. Note that the two first cases in this figure are asymmetric (it can be centered in the pattern or in its antipattern) while the other two are symmetric around $m = 0$, due to the caotic itineracy among all patterns and the particular set of patterns we used.

In order to deepen further on the nature of the chaotic switching, we have computed the normalized power spectra $p(\omega)$ of the time series for the mean firing rate m. If one computes the associated entropy [Cortes *et al.*, 2007], namely, $S = -\sum_{\omega} p(\omega) \log p(\omega)$, it ensues a sharp minimum at $S \simeq 0.37$ for $\Phi = -0.048$ (data not shown). The series corresponding to this minimum and, for comparison purposes, a different one for a much larger entropy are presented in Fig. 7 (left). The power spectra for these two series is presented in Fig. 7(right). This reveals a qualitative change of behaviour, namely, that (only) the series describing a more efficient chaotic mechanism exhibit a power law distribution. We are presently analyzing in more detail this interesting phenomenon.

4 Discussion

We have described in this paper details concerning a model network in which connections are heterogeneously weighted and time–dependent, namely, correlated to the global activity. As documented above, these two conditions occur in many natural networks. Furthermore, only a fraction ρ of nodes are active at each time, so that the rest maintain the previous state. This would occur in an excitable media, for instance.

A main conclusion is that, although the synchronization parameter ρ is generally irrelevant, varying ρ may greatly modify the system behaviour under certain conditions. The necessary condition is a kind of susceptibility

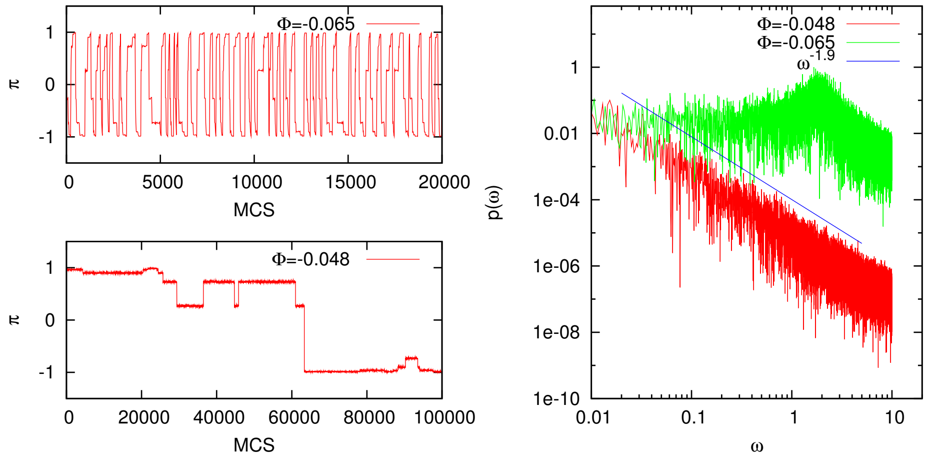

Figure 7: Left: Time series for the overlap π in the case $\rho = 0.632$, $M = 1$, $\beta = \infty$ (zero temperature), $N = 3600$, and $\Phi = –0.048$ (top) and –0.065 (bottom) showing chaotic transitions between the associated pattern and its antipattern. This series correspond to entropies $S \simeq 0.37$ and 0.9, respectively. Left: The power spectra corresponding to the two series in the left panels. The straight line here has slope 1.9.

or sensitivity to external stimuli which greatly favours dynamic instabilities. It may be achieved in our example by appropriate tuning of two parameters, Φ and the inverse temperature β. The former induces either enhancement ($\Phi > 1$) or lowering ($\Phi < 1$) for positive Φ, or even change of sign for negative Φ, of the intensities of connections. This process is a fast one —as compared with the nodes changes—, and it occurs more likely the larger the current degree of order is.

Within the most interesting range of its parameters, our model exhibits heteroclinic trajectories which imply, in particular, a kind of *dynamic association*. That is, the network activity either goes to one attractor for $\rho < \rho_c$, or else, for larger ρ, is capable of an intriguing programme of visits to possible attractors. The dynamics may abruptly become chaotic, which seems the most relevant regime. Besides synchronization of a minimum of nodes, this requires careful tuning of ρ, β and Φ. That is, as suggested by Fig. 2, there is a complex parameter space which makes it difficult to predict the ensuing behaviour for slight changes of parameter values.

The most interesting behaviour of the network consists of *switching* among attractors, that can either be regular for $\rho < \rho_c$, or chaotic which makes the process much more efficient. More specifically, we observe a highly–structured chaotic itinerancy process in which, as illustrated in Fig. 5, modifying ρ within a chaotic window —which requires also tuning β and Φ— one may control the subset of visited attractors. That is, increasing ρ within the relevant regime makes the system to visit *more distant* (less correlated) attractors. In this way the system may perform, for instance, family discrimination and classification by tuning ρ [Cortes *et al.*, 2005]. On the other hand, the complexity of the parameter space for $\rho > \rho_c$ suggest that one could devise a method to control chaos in these cases, and also that one should pay attention to these facts when determining efficient computational strategies in artificial machines. Similar switching phenomena, in which the activity describes a heteroclinic path among saddle states, has already been incorporated in models which thus simulate experiments on animal olfactory systems [Ashwin & Timme, 2005; Rabinovich *et al.*, 2001; Mazor & Laurent, 2005; Huerta & Rabinovich, 2004]. Comparable oscillatory activity has been reported to occur in cultured neural networks [Wagenaar *et al.*, 2006] and ecology models and food webs [Hofbauer & Sigmund, 1989; Vandermeer, 2004; Vandermeer *et al.*, 2007], and it is believed it could account for other natural phenomena as well [Ashwin & Timme, 2005].

Finally, an important feature of the model chaotic itinerancy is illustrated in Fig. 7. This reveals the existence of power–law distributions within the regimes in which the network exhibits its most interesting behaviour. This is the case for the power spectra of time series and for the time spent in the neighbourhood of each attractor for appropriate values of ρ. This fact suggests that a *critical condition* which has been called for to explain some of the brain exceptional behaviour [Eurich *et al.*, 2002; Haldeman & Beggs, 2005; Chialvo, 2006] could perhaps consists, as in our model here, of a highly susceptible, unstable and chaotic condition similar to the one we have described for the model.

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