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Noise Enhanced Metastability in a Nonequilibrium Ferromagnetic System: Mean Field and Computational Study

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We study metastability in a nonequilibrium Ising-like ferromagnetic system, using both mean field theory and simulations. In particular, we pay attention to the intrinsic coercive field, which in this case is the magnetic field separating the metastable region from the unstable one. We find that, under strong nonequilibrium conditions, the intrinsic coercive field exhibits reentrant behavior as a function of temperature. This observation involves the presence of a *non-linear cooperative phenomenon* between the thermal noise and the non-thermal (nonequilibrium) fluctuation source: although both noises add independently disorder to the system, which involves the attenuation or even the destruction of the existing metastable states, the combination of both noise sources not always implies a larger disorder, giving rise to regions in parameter space where there are no metastable states for low and high temperatures, existing however metastability for intermediate temperatures. We argue that the observed noise enhanced metastability may be understood in terms of multiplicative noise.

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INTRODUCTION

The concept of metaestability[1] is a cornerstone in many different branches of Science. Metastability is observed ubiquitously in Nature, and it usually determines the system behavior. Some examples can be found in the Standard Model vacuum[2], quark/gluon plasma[3], superconductors and superfluids[4], electronic circuits[5], globular proteins[6], magnetic systems[7], supercooled liquids[8], climate models [9], black holes and protoneutronic stars[10], cosmology[11], etc. A better microscopic understanding of this ubiquitous phenomenon is then of great theoretical and technological interest, besides being a formidable mathematical challenge.

A related problem of particular importance is that posed by magnetic storage of information. In this case, each individual domain composing a magnetic material is oriented using a strong magnetic field, thus defining a bit of information. A main concern is to retain the domains' orientations for as long as possible in the presence of weak arbitrarily-oriented external magnetic fields. The interaction with these weak external fields often produces metastable states in the domains, and the resistance of stored information depends on the properties of these metastable states, including the details of their decay.

In general, the study of real magnetic systems with many degrees of freedom is a formidably complicated task. One is therefore forced to investigate simplified models of real magnets that, while capturing their relevant ingredients, are much more easily tractable. In this way, there has been in last decades a huge amount of works studying the problem of metastability in lattice models of classical spins. The most studied model has been the Ising model in two and three dimensions.[12, 13, 15–24] The general interest in this model is two-fold. On one hand, it captures many of the fundamental features of a wide class of real systems. On the other hand, many of its equilibrium properties are analytically known in one and two dimensions[14], which makes more easy any theoretical approach to metastability in this model. In this way, continuous theories based on nucleation mechanisms have been proposed which successfully describe the evolution from the metastable state to the stable one.[15, 16] Also the problem of metastability in the low temperature limit has been exactly solved.[17] These theoretical results have been checked many times via computer simulations.[18, 19] Likewise, the effects that open borders[20, 21], quenched impurities[22] and demagnetizing fields[23] have on the properties of metastable states in these systems have also been investigated.

With some exceptions[24], most works on metastability in magnetic systems have been limited to equilibrium models. Although metastability is a dynamic phenomenon not included in the Gibbs formalism,[1] so successful on the other hand when describing equilibrium states, it is possible to understand such phenomenon in equilibrium systems extending dynamically Gibbs theory using the thermodynamic potentials there defined.[15, 16] However, most of the systems we find in Nature are out of equilibrium: they are open systems, subject to thermal or density gradients, mass and/or energy currents, which suffer the action of external agents, contain impurities or are subject to several sources of non-thermal noise, etc. In particular, this is the case for most natural magnetic systems. As an example, it has been observed that some properties of metastable states in certain mesoscopic magnetic particles are highly affected by quantum tunneling of individual spins,[25] which may be thought as a pure nonequilibrium process since it breaks detailed balance. Furthermore, there are nonequilibrium lattice spin models which reproduce some of these results.[24] Hence, if we want to understand metastability in real (i.e., nonequilibrium) magnetic systems we must study simplified nonequilibrium models. For nonequilibrium systems there is no theory equivalent to equilibrium Statistical Mechanics that connects their microscopic properties with their macroscopic phenomenology. Studying metastability in systems far from equilibrium we address the problem of the existence of some nonequilibrium functional controlling the relaxation from the metastable state in a way similar to the equilibrium free energy.[26]

Summing up, in this paper we study metastability in magnetic thin films under nonequilibrium conditions. On the analogy of equilibrium systems, it seems sensible to model these magnetic systems using an (oversimplified) bidimensional kinetic Ising lattice with nearest neighbor interactions and periodic boundary conditions. In addition, we will consider a (very) weak random dynamic perturbation competing with the usual thermal spin flip process. It has been shown that the presence of this weak perturbation could explain some intriguing properties of some real magnetic materials, as for instance the non-vanishing value of magnetic viscosity in the low temperature limit.[24, 25] The impurity makes the system to reach asymptotically a nonequilibrium steady state. That is, we assume that a principal role of the microscopic disorder which is generally present in actual specimens consists in modifying the dynamics -in a way similar to that of an external non-Hamiltonian agent.[27]

It is observed that, under the action of the dynamic perturbation and a weak magnetic field oriented opposite to the initial magnetization, the system is trapped in a metastable state with, where it spends a long time as compared to the typical relaxation time in the system. However, fluctuations make the system to eventually evolve from the metastable well to the stable one, where magnetization is oriented along the external field direction. In this paper we are interested in the effects that nonequilibrium conditions induce on some of the *static* properties associated to metastability: localization of the metastable and stable wells, the onset of instability, etc. In particular, we will pay attention to the magnetic field strength for which the metastable state becomes unstable. This field is known in this case as *intrinsic coercive field*[26**?**], h^*, and it plays a role in magnets equivalent to the spinodal curve for density-conserved systems. The study of h^* will allow us to uncover a non-lineal cooperative phenomenon between the thermal noise and the nonequilibrium fluctuations (parametrized by the dynamic random perturbation) in the strong nonequilibrium regime. In particular, we observe in this regime that while metastable states are not observed in the low temperature limit, they do emerge for intermediate temperatures, thus signaling a *noise enhanced metastability.*

The paper is organized as follows. In Section II we describe our model in detail, sumarizing some of its properties. In Section III we briefly derive a first order dynamic mean field approximation known as Pair Approximation[27, 28], as applied to our ferromagnetic model. Section IV is devoted to some results derived from this approach for the static properties associated to metastability in this system. In particular, in this section we evaluate the intrinsic coercive field in mean field theory. In Section V we introduce a method to measure the intrinsic coercive field in Monte Carlo simulations of the real system. There we also compare our measurements with the predicted mean field result. Finally, Section VI presents our conclusions, paying special attention to the physical mechanism responsible of the observed cooperative phenomenon between thermal and non-thermal noises.

THE MODEL

The two-dimensional Ising model[29] is defined on a square lattice $\Lambda = \{1, \ldots, L\}^2 \subset \mathbb{Z}^2$ of side L. On each lattice node a spin variable is defined, s_i, with $i \in [1, N]$, $N = L^2$. Each spin can take two different values, $s_i = \pm 1$. The system is characterized by the Hamiltonian,

$$\mathcal{H}(\mathbf{s}) = -J \sum_{\langle i,j \rangle} s_i s_j - h \sum_{i=1}^{N} s_i \qquad (1)$$

where $J > 0$ is the (ferromagnetic) coupling constant, $\mathbf{s} \equiv \{s_i, i = 1, \ldots, N\}$ is the system's configuration, and h is an external magnetic field. The first sum runs over all nearest neighbor pairs, $\langle i, j \rangle$, while the second sum runs over all spins. We endow this kinetic model with a single spin flip dynamics determined by the following transition rate, consequence of the superposition of two "canonical" drives,

$$\omega(\mathbf{s} \to \mathbf{s}^i) = p + (1-p) \frac{\mathrm{e}^{-\beta \Delta \mathcal{H}(s_i, n_i)}}{1 + \mathrm{e}^{-\beta \Delta \mathcal{H}(s_i, n_i)}} \qquad (2)$$

Here $\mathbf{s}^i$ stands for the configuration $\mathbf{s}$ after flipping the spin at node i, $\beta = 1/T$ is the inverse temperature, and $\Delta \mathcal{H}(s_i, n_i) \equiv \mathcal{H}(\mathbf{s}^i) - \mathcal{H}(\mathbf{s}) = 2 s_i [2J(n_i - d) + h]$, where $n_i \in [0, 4]$ is the number of up nearest neighbors of the spin at node i, and d is the system dimension. We have fixed Boltzmann constant to unity.

One can interpret the above dynamical rule as describing a spin flip process under the action of two competing heat baths: with probability p the spin flip is performed completely at random, independently of any energetic consideration (we can interpret in this case that $\mathbf{s}$ is in contact with a heat bath at *infinite* temperature), while the spin flip is performed at temperature T (via the usual Glauber rate) with probability $(1-p)$. The dynamics we have chosen is a particular case of the general class of competitive dynamics.[30] Any competitive transition rate as the one written in (2) will produce in the system what is called in literature *dynamical frustration*,[30–32] and the competition between both dynamics generically drives the system towards a nonequilibrium steady state.[33] Moreover, the selected dynamical rule, eq. (2), rests on realistic grounds. In fact, Glauber dynamics can be derived from first principles for a system of $\frac{1}{2}$-spin fermionic quantum particles, each one subject to its own thermal bath.[34] On the other hand, the weak dynamic perturbation parameterized by p emulates in a vague sense the effect of quantum tunneling of individual spins in real magnetic systems. In fact, the existence of this small $p \neq 0$ allows the spins to flip independently of any energetic constraint imposed by their surroundings with a (very) low probability. This is roughly what quantum tunneling produces in real spins: the spin is able to flip by *tunneling* through the energy barrier which impedes its classical (thermal) flipping. We also can interpret in a more general way the dynamic random perturbation parameterized by p as a generic source of disorder and randomness, i.e. as a simplified representation of the impure dynamic behavior typical of real systems.[24]

For $p = 0$, the rate (2) corresponds to the canonical Ising case which converges asymptotically towards a Gibbs equilibrium state at temperature T and energy $\mathcal{H}$. In this case the model for $h = 0$ exhibits a second order phase transition at a critical temperature $T = T_c \approx 2.2691J \equiv T_{ons}$.[14] For $p \neq 0$ the conflict in (2) impedes canonical equilibrium, and as we mentioned above the system then evolves towards a nonequilibrium steady state whose nature essentially differs from a Gibbs state at temperature T. The system now, and always for $h = 0$, exhibits a second order phase transition at a critical temperature $T_c(p) < T_{ons}$ for small enough values of p. This critical point, which belongs to the Ising universality class[35, 36], disappears for values of p above certain critical value p_c.

Metastable states are characterized in general by their long lifetime. This long decay time impedes in some cases the application of straightforward Monte Carlo schemes[37] when simulating the system metastable behavior. Therefore, we need to use in some cases advanced, faster-than-real-time algorithms which deal with the above problem. Hence, whenever necessary, we have used in this paper the *Monte Carlo with Absorbing Markov Chains* (MCAMC) algorithm [38, 39], together with the so-called *slow forcing approximation*.[40]

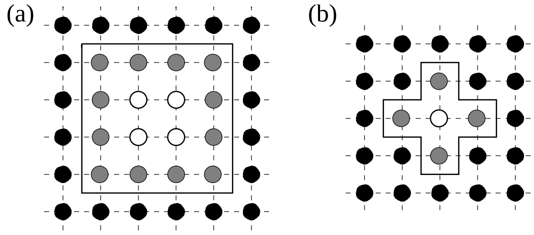

FIG. 1: Different examples of spin domains, each one characterized by a different kind of partition $\mathbb{P}(\Lambda)$. External spins are coloured in black, surface spins are gray, and internal spins are white.

FORMULATION OF THE PAIR APPROXIMATION

The approach we here describe, following references [26, 30], is a generalization for the study of dynamic problems of Kikuchi's method[41] known as *Cluster Variation Method*. This method has been reformulated for the study of some nonequilibrium systems by Dickman and other authors[27, 28], with the name of Pair Approximation. It is a mean field approximation as far as it neglects correlations actually present in the system, and it builds, using this assumption, a set of equations for averaged observables which describe the dynamical and statical behavior of the system.

Our starting point is the master equation, which governs the system dynamics,

$$\frac{\mathrm{d}P(\mathbf{s};t)}{\mathrm{d}t} = \sum_{i\in\Lambda}\Big[\omega(\mathbf{s}^i \to \mathbf{s})P(\mathbf{s}^i;t) - \omega(\mathbf{s}\to\mathbf{s}^i).P(\mathbf{s};t)\Big] \tag{3}$$

Here $P(\mathbf{s};t)$ is the probability of finding the system in a state $\mathbf{s}$ for time t, and $\omega(\mathbf{s}\to\mathbf{s}^i)$ is the above-described transition rate. Let's assume now that we perform a partition $\mathbb{P}$ of the lattice Λ, in such a way that domains q_j resulting from this partition will verify the following restrictions: $q_j \in \mathbb{P}(\Lambda)$ such that $q_j \cap q_{j'} = \varnothing$ if $j \neq j'$ and $\bigcup_j q_j = \Lambda$. Subindex j indicates the domain lattice position. Given a domain q_j, its surface $\mathcal{S}_j$ is formed by all spins in the domain which have some nearest neighbor outside the domain. Equivalently, the domain's interior, $\mathcal{I}_j$, is formed by all spins in the domain whose nearest neighbor spins are also inside the domain.[42] Thus $q_j = \mathcal{I}_j \cup \mathcal{S}_j$. Fig. 1 shows an example. Let's assume now that we have a local observable $A(\mathbf{s}_{q_j};j)$ which exclusively depends on spins belonging to domain q_j (we denote these spins as $\mathbf{s}_{q_j}$). The average of this observable at time t is,

$$\langle A(j)\rangle_t = \sum_{\mathbf{s}} A(\mathbf{s}_{q_j};j)P(\mathbf{s};t) \tag{4}$$

Using eqs. (3) and (4) we obtain a temporal evolution equation for the average,[26]

$$\frac{\mathrm{d}\langle A(j)\rangle_t}{\mathrm{d}t} = \sum_{\mathbf{s}} \sum_{i\in q_j} \Delta A(\mathbf{s}_{q_j};j;i)\omega(\mathbf{s}\to\mathbf{s}^i)P(\mathbf{s};t) \quad (5)$$

where we define $\Delta A(\mathbf{s}_{q_j};j;i) = A(\mathbf{s}^i_{q_j};j) - A(\mathbf{s}_{q_j};j)$ We can rewrite eq. (5) taking into account the definition of surface and interior of domain q_j,

$$\begin{aligned}\frac{\mathrm{d}\langle A(j)\rangle_t}{\mathrm{d}t} &= \sum_{\mathbf{s}_{q_j}} \sum_{i\in\mathcal{I}_j} \Delta A(\mathbf{s}_{q_j};j;i)\omega(\mathbf{s}_{q_j}\to\mathbf{s}^i_{q_j})Q(\mathbf{s}_{q_j};t) \\ &+ \sum_{\mathbf{s}} \sum_{i\in\mathcal{S}_j} \Delta A(\mathbf{s}_{q_j};j;i)\omega(\mathbf{s}\to\mathbf{s}^i)P(\mathbf{s};t) \quad (6)\end{aligned}$$

where we have defined the projected probability,

$$Q(\mathbf{s}_{q_j};t) = \sum_{\mathbf{s}-\mathbf{s}_{q_j}} P(\mathbf{s};t) \quad (7)$$

which is the probability of finding domain q_j in a configuration $\mathbf{s}_{q_j}$ at time t. Writing $\omega(\mathbf{s}_{q_j}\to\mathbf{s}^i_{q_j})$ in the first term of right hand side in eq. (6) we stress the fact that the probability of flipping a spin in the domain's interior depends exclusively on the spins belonging to this domain.

As a first approximation, we assume from now on that our system is *homogeneous*, i.e. its properties do not depend on the selected point in the system. Hence $\langle A(j)\rangle \equiv \langle A\rangle$, $q_j\equiv q$, $\mathcal{I}_j\equiv\mathcal{I}$ and $\mathcal{S}_j\equiv\mathcal{S}$. Equivalently, we suppose that the partition is regular, so all domains are topologically identical. On the other hand, eq. (6) shows two well-differentiated terms. The first one only depends on what happens in the domain interior, while the second one involves the domain's surface, couples the domain dynamics with its surroundings, and makes the problem unapproachable in practice. Our second approximation consists in neglecting the surface term in this equation. This approximation involves that the domain is *kinetically isolated* from the exterior[30]: the domain's exterior part does not induce any *net* variation on the local observables defined inside the domain. Thus we are neglecting in practice correlations larger than the domain size. Under both *homogeneity* and *kinetic isolation* approximations, the equation we must study reduces to,

$$\frac{\mathrm{d}\langle A\rangle_t}{\mathrm{d}t} = \sum_{\mathbf{s}_q} \sum_{i\in\mathcal{I}} \Delta A(\mathbf{s}_q;i)\omega(\mathbf{s}_q\to\mathbf{s}^i_q)Q(\mathbf{s}_q;t) \quad (8)$$

In order to go on, we must know the expression for the projected probability $Q(\mathbf{s}_q;t)$. This probability can be decomposed in terms of $n-$body correlation functions[26, 30]. Hence, in order to be coherent with the kinetic isolation approximation, which neglects long range correlations, we express the probability $Q(\mathbf{s}_q;t)$ as a function of a reduced number of correlation functions. In particular, our third approximation consists in expressing all correlations as functions of magnetization $\langle s\rangle$ and the nearest neighbors correlation function, $\langle s_i s_j\rangle$, with i and j nearest neighbors sites inside the domain. This is equivalent to writing $Q(\mathbf{s}_q;t)$ as a function of $\rho(s,s')$, which is the density of (s,s') nearest neighbors pairs, and as a function of the density of s spins, $\rho(s)$. We only have to define now the domain q that we are going to use in our study. Since we only take into account nearest neighbors correlations, we must choose a domain with only one spin in its interior, and $2d$ spins (the nearest neighbors of the interior spin) on the surface, being d the system dimension (in our particular case, $d=2$). Fig. 1.b shows an example of this domain type.

The probability of finding this domain in a configuration defined by a central spin s and n up nearest neighbors can be easily written,

$$Q(\mathbf{s}_q;t)\equiv Q(s,n) = \binom{2d}{n}\rho(s)^{1-2d}\rho(+,s)^n\rho(-,s)^{2d-n} \quad (9)$$

Taking into account that $\rho(+,-)=\rho(-,+)=\rho(+)-\rho(+,+)$ and $\rho(-,-)=1+\rho(+,+)-2\rho(+)$, and denoting $x\equiv\rho(+)$ and $z\equiv\rho(+,+)$, we can write eq. (8) as,

$$\begin{aligned}\frac{\mathrm{d}\langle A\rangle_t}{\mathrm{d}t} &= \sum_{n=0}^{2d}\binom{2d}{n}\Big[\Delta A(+,n)x^{1-2d}z^n(x-z)^{2d-n}\omega(+,n) \\ &-\Delta A(-,n)(1-x)^{1-2d}(x-z)^n(1+z-2x)^{2d-n}\omega(-,n)\Big] \quad (10)\end{aligned}$$

where $\omega(\mathbf{s}_q\to\mathbf{s}^i_q)\equiv\omega(s,n)$, and we have restricted to local *isotropic* observables, i.e. to observables $A(\mathbf{s}_q;t)$ which depend on $\mathbf{s}_q$ through the pair (s,n), so that $A(\mathbf{s}_q;t)\equiv A(s,n)$.

We now write down two local microscopic observables, $A_1(s,n)$ and $A_2(s,n)$, such that their configurational averages correspond to x and z, respectively. We can check that these observables are,

$$\begin{aligned}A_1(s,n) &= \frac{1+s}{2} \\ A_2(s,n) &= \frac{n}{2d}\times\frac{1+s}{2} \quad (11)\end{aligned}$$

Hence, $\langle A_1(s,n)\rangle = x$ and $\langle A_2(s,n)\rangle = z$. From these expressions we can see that $\Delta A_1(s,n)=-s$ and $\Delta A_2(s,n)=-sn/2d$. Applying eq. (10) to both observables we find,

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= -\sum_{n=0}^{2d}\binom{2d}{n}\Big[x^{1-2d}z^n(x-z)^{2d-n}\omega(+,n) \\ &- (1-x)^{1-2d}(x-z)^n(1+z-2x)^{2d-n}\omega(-,n)\Big] \quad (12) \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= -\frac{1}{2d}\sum_{n=0}^{2d}\binom{2d}{n}n\Big[x^{1-2d}z^n(x-z)^{2d-n}\omega(+,n) \\ &- (1-x)^{1-2d}(x-z)^n(1+z-2x)^{2d-n}\omega(-,n)\Big] \quad (13)\end{aligned}$$

These two equations are the basic equations in Pair Approximation. Hence, once defined the transition rate

$\omega(s,n)$ (see eq. (2)), the general working method thus consists in calculating both $x(t)$ and $z(t)$ using the above equations, and using eq. (10) and the results for $x(t)$ and $z(t)$ calculate any other local magnitude.

STATIC PROPERTIES

In a first step we can study the stationary solutions of eqs. (12) and (13) as well as their stability for the nonequilibrium ferromagnetic system. We have two non-linear coupled differential equations,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F_1(x,z)$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = F_2(x,z) \qquad (14)$$

where $F_1(x,z)$ and $F_2(x,z)$ are defined by eqs. (12) and (13), respectively, once we include in these equations the explicit form of the transition rate, eq. (2). The stationary solutions of the previous coupled set of equations, x_{st} and z_{st}, are the solutions of the system,

$$F_1(x_{st},z_{st}) = 0 \quad , \quad F_2(x_{st},z_{st}) = 0 \qquad (15)$$

Both stable and metastable states in a generic system are *locally stable* under small perturbations. Hence we are interested in locally stable stationary solutions of this set of equations. In order to stablish a local stability criterion, we perturb the steady solutions, $x = x_{st} + \epsilon_x$, $z = z_{st} + \epsilon_z$, with $\epsilon_x, \epsilon_z \ll 1$, and analyze the time evolution of the perturbed state. This standard analysis yields the conditions,

$$\left(\frac{\partial F_1}{\partial x}\right)_{st} + \left(\frac{\partial F_2}{\partial z}\right)_{st} < 0 \qquad (16)$$
$$\left(\frac{\partial F_1}{\partial x}\right)_{st}\left(\frac{\partial F_2}{\partial z}\right)_{st} - \left(\frac{\partial F_1}{\partial z}\right)_{st}\left(\frac{\partial F_2}{\partial x}\right)_{st} > 0$$

This criterion, known as Hurwitz criterion[46], states the necessary and sufficient conditions that a steady solution of our set of non-linear differential equations must fulfill in order to be locally stable under small perturbations.

Phase Diagram

We are also interested in simple necessary (although not sufficient) conditions that locally stable steady states must fulfill. For instance, if we perturb the stationary state by only varying x and keeping untouched z, that is, $x = x_{st} + \epsilon_x$ and $z = z_{st}$, with $\epsilon_x \ll 1$, we arrive to the following solution once we apply standard stability analysis,

$$\epsilon_x(t) \approx \epsilon_x^0 \mathrm{e}^{t(\frac{\partial F_1}{\partial x})_{st}} \qquad (17)$$

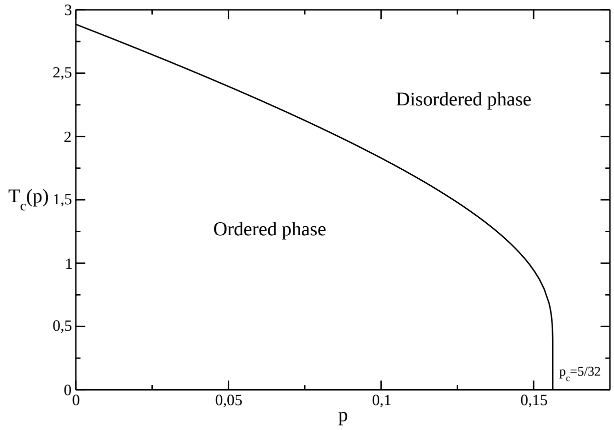

FIG. 2: Critical temperature for the nonequilibrium ferromagnetic system as a function of p in Pair Approximation

so the steady state defined by (x_{st}, z_{st}) *can* be locally stable only if $(\frac{\partial F_1}{\partial x})_{st} < 0$. It will be unstable if this derivative is larger than zero. The condition

$$\left(\frac{\partial F_1(x,z)}{\partial x}\right)_{st} = 0 \qquad (18)$$

defines a point (x_{st}^c, z_{st}^c) of incipient instability or marginal stability which signals the presence of an underlying critical point or second order phase transition for $h = 0$ between a disordered phase and an ordered phase.[26, 30] Just at this critical point we have $x_{st}^c = \frac{1}{2}$, since it separates an ordered phase with non vanishing spontaneous magnetization from a disordered phase with zero spontaneous magnetization. This observation trivially implies $z \equiv \rho(+,+) = \rho(-,-) \equiv (1 + z - 2x)$ at the critical point. We also have $z_{st}^c = \frac{1}{3}$ at the critical point.[47] Using these values for x_{st}^c y z_{st}^c in eq. (18) once we substitute there the explicit form of $F_1(x,z)$, eq. (12), and solving for temperatures, we find,

$$\frac{T_c(p)}{J} = \frac{-4}{\ln\left[-\frac{1}{2} + \frac{3}{4}\sqrt{\frac{1-4p}{1-p}}\right]} \qquad (19)$$

This equation yields the critical temperature for the nonequilibrium model in first order mean field approximation as a function of parameter p, which characterizes the dynamic nonequilibrium perturbation present in the system. We can also derive this expression from eqs. (17), which define the general stability criterion, applying the marginal stability condition. Fig. 2 shows $T_c(p)$ as a function of p. For $p = 0$ the critical temperature $T_c(p)$ is just the Bethe temperature, $T_{Bethe}/J \approx 2.8854$, to be compared with the exact critical value for $p = 0$, which is the Onsager temperature, $T_{ons}/J \approx 2.2691$. For each value of p, temperature $T_c(p)$ signals the border, always in mean field approximation, between the ordered phase at low temperatures ($T < T_c(p)$) and the disordered phase at higher temperatures ($T > T_c(p)$). There

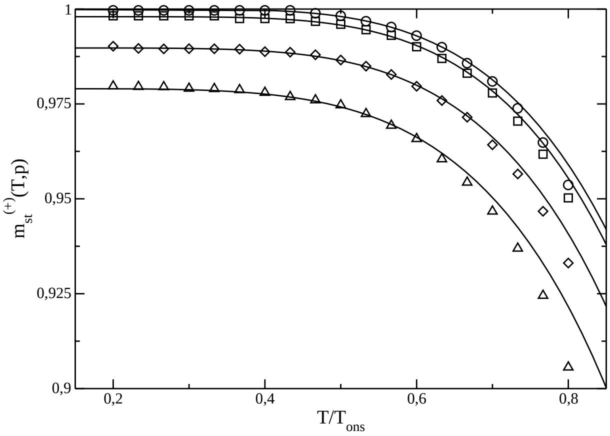

FIG. 3: Locally stable steady state magnetization as a function of temperature (in units of Onsager temperature) for different values of p and for $h = 0$. In particular, from top to bottom, $p = 0$, 0.001, 0.005 and 0.01. Points are results obtained from Monte Carlo simulations for a system with $L = 53$. Continuous lines are the solutions in Pair Approximation. Error bars in computational results are much smaller than the symbol sizes.

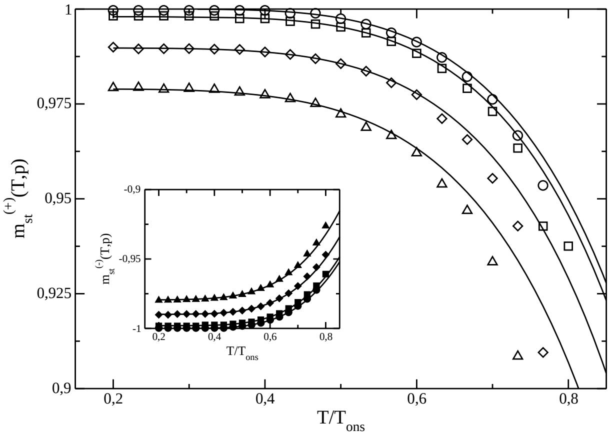

FIG. 4: Magnetization of the locally stable steady state of positive magnetization as a function of temperature (in units of Onsager temperature) for different values of p and $h = -0.1$. From top to bottom, $p = 0$, 0.001, 0.005 and 0.01. Points are results obtained from Monte Carlo simulations for a system of size $L = 53$. The continuous lines are Pair Approximation solutions. Error bars associated to computational results are much smaller than symbol sizes. In the inset we show the results for the negative magnetization branch.

is a critical value of p, p_c, such that for larger values of p there is no ordered phase for any temperature. This value p_c can be obtained from the condition $T_c(p_c) = 0$, yielding $p_c = \frac{5}{32} = 0.15625$. On the other hand, the phase transition we obtain in mean field approximation obviously belongs to the mean field universality class, on the contrary to the real nonequilibrium system, which belongs to the Ising universality class.

Stable and Metastable States

After this brief parenthesis about the model critical behavior, we turn back to study its locally stable steady states in the ordered phase. These stationary states (x_{st}, z_{st}) will be given by solutions of the set of non-linear differential equations (15), subject to the Hurwitz local stability condition, eqs. (17). Unfortunately, the non-linearity of the set of eqs. (15) impedes any analytical solution, so we have to turn to numerical solutions.

In a first step we center our attention on the study of stationarity for zero magnetic field, $h = 0$. In this case, the system exhibits up-down symmetry, so we will have two symmetrical branches of solutions in the ordered phase, one of positive magnetization and another one with negative magnetization. Moreover, we can prove for $h = 0$ that if the pair (x_{st}, z_{st}) is a locally stable steady solution of the set of eqs. (15), then the pair $(1 - x_{st}, 1 + z_{st} - 2x_{st})$ is also a locally stable steady solution. If we solve the set of eqs. (15) using standard numerical techniques[48] and we keep only those solutions which fulfill Hurwitz criterion, we finally obtain the results shown in Fig. 3. There we compare the theoretical predictions for the positive magnetization branch with results obtained from Monte Carlo simulations for different values of the dynamic random perturbation p. The agreement between theory and computational results is excellent for low and intermediate temperatures for all studied values of p, failing gradually as we approach the critical temperature. Furthermore, the differences between theory and simulations begin to be relevant for temperatures higher than a 75% of the critical temperature for each case. Such inaccuracy of Pair Approximation for temperatures close enough to the critical one was expected a priori, since mean field theory neglects long range correlations, which on the other hand gradually arise as we approach the critical region. Fig. 3 shows also that, as we increase p for a fixed temperature, the system's magnetization decrease in absolute value. Therefore, an increase of p is equivalent to an increase of disorder in the system. On the other hand, the qualitative form of the curve $m_{st}^{(+)}(T,p)$ does not change for $p \neq 0$ as compared to the equilibrium system ($p = 0$).

The Monte Carlo simulations whose results are shown in Fig. 3 have been performed for a system with size $L = 53$, subject to periodic boundary conditions, with $h = 0$ and different values of T and p. In order to measure the magnetization of the positive magnetization steady state, we put the system in an initial state with all spins up. We let evolve this state with the dynamics (2) for certain values of T and p. After some relaxation time, the systems starts fluctuating around the steady state. We then measure magnetization at temporal intervals Δt

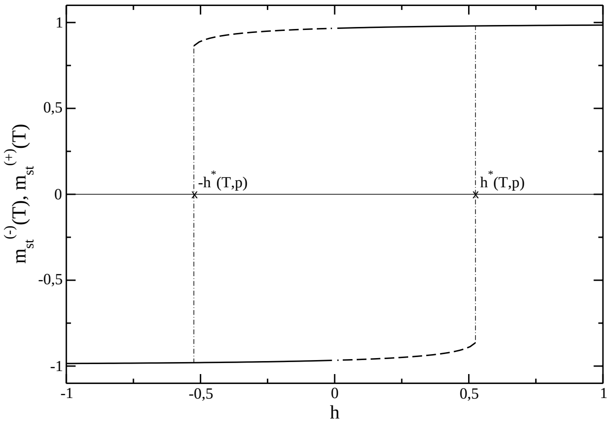

FIG. 5: Locally stable steady state magnetization for both magnetization branches as a function of magnetic field h for fixed $T = 0.7T_{ons}$ and $p = 0.005$. The continuous line represents stable states, the dashed line represents metastable states, and the dot-dashed line signals the discontinuous transition where metastable states disappear. This discontinuity appears for a magnetic field $h^*(T,p)$.

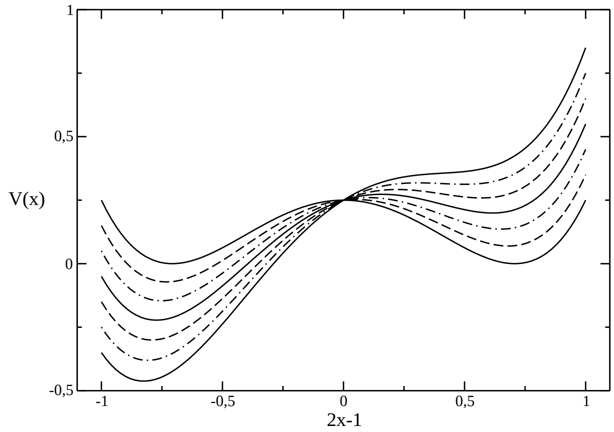

FIG. 6: Schematic plot of the potential $V(x)$ defined in the main text, for fixed temperature and p, and for several different values of magnetic field $h < 0$. Notice that the local minimum in the positive magnetization branch is attenuated as $|h|$ increases, up to its disappearance for large enough values of $|h|$.

larger than the correlation time, and we average over different measurements. The error associated to this average is the standard statistical error. A second method to measure the stationary state magnetization is based on the stable phase growth and shrinkage rates[38], which we will define later on. Both methods yield equivalent results.

We also can study the steady states for $h < 0$. In particular, here we study the case $h = -0.1$. As opposed to the $h = 0$ case, now there is no up-down symmetry. Therefore the negative and positive magnetization branches are here different. Moreover, the locally stable steady state with positive magnetization is now metastable. Numerically solving the set of eqs. (15) subject to the conditions (17) we obtain the results shown in Fig. 4. In this figure we also show results from simulations analogous to the ones described above, but with $h = -0.1$, and where the initial state is defined with all spins up (down) if we want to measure the positive (negative) magnetization branch. Comparatively, these results are very similar in spirit to the results obtained for $h = 0$.

Hysteresis and the Intrinsic Coercive Field

An interesting question consists in knowing what happen to locally stable steady states as we change the magnetic field. In order to answer this question we numerically solve again the set of eqs. (15) subject to Hurwitz conditions for fixed temperature and dynamic random perturbation p, varying the magnetic field between $h = -1$ and $h = +1$. In particular, Fig. 5 shows the result for $T = 0.7T_{ons}$ and $p = 0.005$. This curve forms what is generally known as a hysteresis loop. Hysteresis is a property of many systems near a first order critical point, and it is intimately related to metastability. A system is said to exhibit *hysteresis* if its properties depend on its previous history. Thus systems showing hysteresis are systems with *memory*.

We observe in Fig. 5 that there is a magnetic field $h^*(T,p) > 0$ such that for all $|h| > h^*(T,p)$ metastable states disappear. This magnetic field $h^*(T,p)$ is known as *intrinsic coercive field*.[49] As we increase the absolute value of the field, metastable states get weaker and weaker. The reason underlies on the increase of the transition rate for spins in the metastable phase as we increase the magnetic field strength, see eq. (2). Thus there is a value of the magnetic field for which the metastable state is no more metastable and transforms into an unstable state. Let's assume that we are able to simplify eqs. (14) in such a way that we know $z = z(x)$. Now we can rewrite eq. (12) as,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\delta V(x)}{\delta x} \tag{20}$$

where $V(x)$ is a (nonequilibrium) potential which controls the system evolution. Fig. 6 shows a schematic plot of this potential for the ordered phase at fixed temperature and p, and for several negative magnetic fields of increasing absolute value. The effect of the magnetic field is to attenuate the local minimum associated to the metastable state. For magnetic fields $|h| < h^*(T,p)$ this local minimum, although attenuated, exists. However, for magnetic fields $|h| > h^*(T,p)$ the metastable minimum disappears, and so the metastable state. Therefore, for $|h| > h^*(T,p)$ the set of eqs. (15) has only one solution, with magnetization sign equal to that of the applied field.

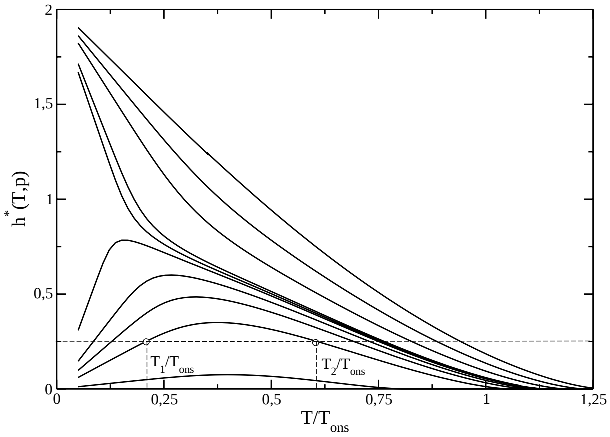

FIG. 7: Intrinsic coercive field, $h^*(T,p)$, as a function of temperature for different values of p. From top to bottom, $p=0$, 0.01, 0.02, 0.03, 0.031, 0.032, 0.035, 0.04, 0.05 and 0.1. Notice that the qualitative change of behavior in the low temperature limit appears for $p \in (0.031, 0.032)$. Here we also show, for $|h|=0.25$, the temperatures $T_1 < T_2$ such that for $T_1 < T < T_2$ there are metastable states for $p=0.05$.

In order to calculate $h^*(T,p)$ we study how a metastable state changes under small perturbations of magnetic field. Let's assume then that $(x_{st}^{h_0}, z_{st}^{h_0})$ is a locally stable stationary state for parameters T, p and h_0, with magnetization opposed to the external magnetic field. If we slightly perturb this magnetic field, $h = h_0 + \delta h$, also the locally stable stationary solution will be modified, $x_{st}^{h} = x_{st}^{h_0} + \epsilon_x$ and $z_{st}^{h} = z_{st}^{h_0} + \epsilon_z$. Applying eqs. (14) to both x_{st}^h and z_{st}^h, and taking into account that they are also steady solutions, we obtain for ϵ_x,

$$\epsilon_x = \Big[\frac{\frac{\partial F_2}{\partial h}\frac{\partial F_1}{\partial z} - \frac{\partial F_1}{\partial h}\frac{\partial F_2}{\partial z}}{\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial z} - \frac{\partial F_2}{\partial x}\frac{\partial F_1}{\partial z}}\Big]_{x_{st}^{h_0}, z_{st}^{h_0}, h_0, T, p} \delta h \tag{21}$$

This equation says that the metastable state magnetization response after a small variation of the magnetic field is proportional to such perturbation in a first approximation. However, the magnetization response will be divergent when,

$$\Big[\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial z} - \frac{\partial F_2}{\partial x}\frac{\partial F_1}{\partial z}\Big]_{x_{st}^{h_0}, z_{st}^{h_0}, h_0, T, p} = 0 \tag{22}$$

When this condition holds, there will be a discontinuity in the metastable magnetization as a function of h. For fixed T and p we thus identify the magnetic field h_0 for which condition (22) is fulfilled as the *intrinsic coercive field*, $h^*(T,p)$. Unfortunately, we cannot analytically calculate $h^*(T,p)$, since we do not explicitely know the metastable solutions $x_{st}^{h_0}$ and $z_{st}^{h_0}$. Solving again the problem with standard numerical methods, we obtain the results shown in Fig. 7. There we plot $h^*(T,p)$ as a function of temperature for different values of the nonequilibrium parameter p. The first conclusion we draw from this family of curves is the existence of two different low temperature limits for $h^*(T,p)$, depending on the value of p. For small enough values of p (including the equilibrium case, $p=0$), the curve $h^*(T,p)$ extrapolates towards 2 in the limit $T \to 0$. In particular this is true for $p \in [0, 0.031]$ (see Fig. 7). On the contrary, for large enough values of p, namely $p \in [0.032, \frac{5}{32})$, the curve $h^*(T,p)$ extrapolates towards 0 in the limit $T \to 0$. There is a critical value for p, that we estimate here to be $\pi_c \approx 0.0315$, which separates both types of asymptotic behaviors.

As we said before, the intrinsic coercive field $h^*(T,p)$ signals the magnetic field strength above which there are no metastable states. As we see in Fig. 7, for $p < \pi_c$ the behavior of $h^*(T,p)$ for the nonequilibrium system is qualitatively similar to that of the equilibrium one: $h^*(T, p < \pi_c)$ is a monotonously decreasing function of T. Therefore, for $p < \pi_c$, if we cool the system we need a stronger magnetic field in order to *destroy* the metastable state. This result agrees with intuition. In a metastable state there are two competing processes: a net tendency of the system to line up in the direction of the field, and a net tendency in order to maintain the spin order, i.e. in order to keep all spins oriented in the same direction (whatever this direction is). A metastable state survives a long time because the tendency towards maintaining the order in the system overcomes the tendency to line up along the field direction. Both the temperature T and the nonequilibrium parameter p are ingredients which introduce disorder in the system. Hence, if we drop temperature, since in this way order grows in the system, we would expect in this phenomenologic picture that the magnetic field needed to *destroy* the metastable state should be stronger, as we effectively check for $p < \pi_c$. In the same way, as p is increased, disorder grows in the system, so $h^*(T,p)$ must decrease for a fixed temperature, as we again observe.

On the contrary, for $p > \pi_c$ the system exhibits an unexpected behavior, difficult to understand using the above phenomenologic picture. Let's assume we fix the magnetic field to be $|h| = 0.25$ and the nonequilibrium parameter to be $p = 0.05 > \pi_c$. As we can see in Fig. 7, we can define two different temperatures, $T_1 < T_2$, such that if $T < T_1$ or $T > T_2$ the system does not exhibit metastable states, while metastable states do exist if temperature lies in the interval $T \in (T_1, T_2)$. The fact that $h^*(T,p)$ extrapolates to zero in the low temperature limit for $p = 0.05 > \pi_c$ points out that the nonequilibrium parameter $p = 0.05$, which is the relevant source of disorder and randomness at low temperatures, takes a value in this case large enough in order to *destroy* on its own any metastable state. In principle, following the above phenomenologic picture, we would say that increasing in this case temperature the metastable state should not ever ex-

Class	Central spin	Number of up neighbors	$\Delta\mathcal{H}$
1	+1	4	8J+2h
2	+1	3	4J+2h
3	+1	2	2h
4	+1	1	-4J+2h
5	+1	0	-8J+2h
6	-1	4	-8J-2h
7	-1	3	-4J-2h
8	-1	2	-2h
9	-1	1	4J-2h
10	-1	0	8J-2h

TABLE I: Spin classes for the two-dimensional isotropic Ising model with periodic boundary conditions. The last column shows the energy increment involved by a spin flip for each class.

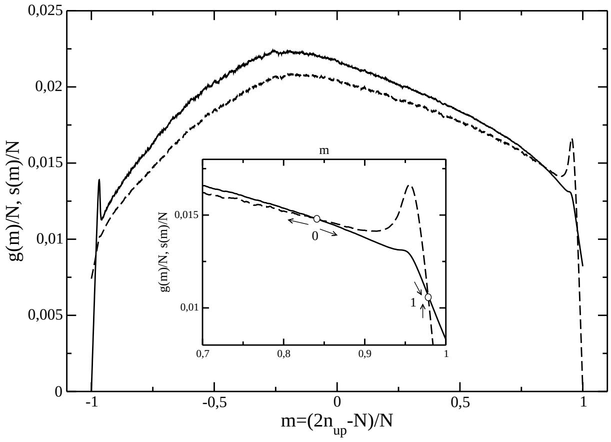

FIG. 8: Growth and shrinkage probabilities of the stable phase, $g(m)/N$ and $s(m)/N$ respectively, for a system of size $L = 53$, with $T = 0.6T_{ons}$, $p = 0.005$ and $h = -0.1$. The continuous line represents $g(m)/N$, while the dashed line represents $s(m)/N$. The inset shows a detail of the positive magnetization (i.e. metastable, $h < 0$) region.

ist, because we add disorder to the system. However, we observe that a regime of intermediate temperatures exists, $T \in (T_1, T_2)$, where metastable states emerge. This observation involves the presence of a *non-linear cooperative phenomenon* between the thermal noise (parameterized by T) and the non-thermal fluctuation source (parameterized by p): although both noises add independently disorder to the system, which involves the attenuation or even the destruction of the existing metastable states, the combination of both noise sources, parameterized in the dynamics (2), not always implies a larger disorder, giving rise to regions in parameter space (T, p) where there are no metastable states for low and high temperatures, existing however metastability for intermediate temperatures. This counter-intuitive behavior resembles in some sense the reentrant behavior of some systems under the action of multiplicative noise, as for instance the annealed Ising model[50], where a disordered phase exists for low and high temperatures, but there is an ordered phase for intermediate temperatures.[51] As we will show below, arguments can be developed pointing out that multiplicative noise is also at the roots of the observed noise enhanced metastability in our nonequilibrium ferromagentic model.

Intrinsic Coercive Field from Monte Carlo Simulations: Stable Phase Growth and Shrinkage Rates

Now we want to check the above mean-field theoretical prediction via computer simulations, so we need to discern when the system exhibits a metastable state. In order to establish a criterion, we must introduce the concept of spin class. For a spin s in the lattice, the spin class to which this spin belongs to is defined once we know the spin orientation, $s = \pm 1$, and its number of up nearest neighbors, $n \in [0, 4]$. Therefore, for the two-dimensional isotropic ($J_x = J = J_y$) Ising model subject to periodic boundary conditions there are 10 different spin classes, schematized in Table I. All spins belonging to the same spin class involve the same energy increment $\Delta\mathcal{H}(s, n)$ when flipped (see Table I), so the transition rate for a spin to flip depends exclusively on the spin class $i \in [1, 10]$ to which the spin belongs to, $\omega_i \equiv \omega(s, n)$, see eq. (2). If $n_k(m)$ is the number of spins in the system that belong to class k when the system has magnetization m, then $n_k(m)\omega_k$ will be the number of spins in class k which flip per unit time when we have $n_{up} = N(1 + m)/2$ up spins. Since in our convention (see Table I) all classes $k \in [1, 5]$ are characterized by a central spin with $s = +1$ and $n = 4, 3, \ldots, 0$ up nearest neighbors, the number of up spins which flip per unit time when magnetization is m will be,

$$g(m) = \sum_{k=1}^{5} n_k(m)\omega_k \tag{23}$$

Since we assume $h < 0$, this observable is the *stable phase growth rate*, and it depends on system's magnetization. In a similar way, we define the *stable phase shrinkage rate*,

$$s(m) = \sum_{k=6}^{10} n_k(m)\omega_k \tag{24}$$

Now $s(m)$ is the number of down spins which flip per unit time when system's magnetization is m.[43]

If we have a state with magnetization m, the rate of change of magnetization will be,

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{2}{N}\left[s(m) - g(m)\right] \tag{25}$$

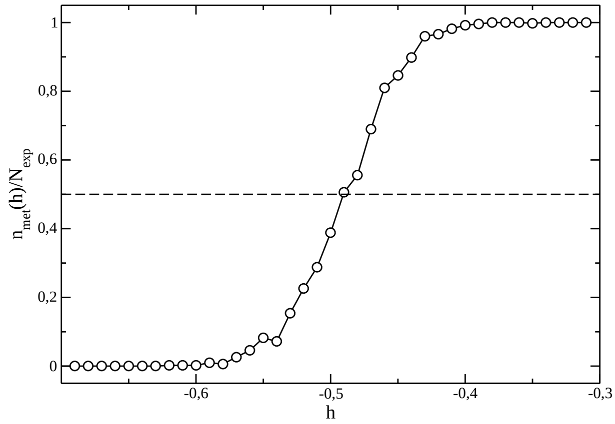

FIG. 9: Probability of finding a metastable state, as defined in the main text, as a function of magnetic field $h < 0$ for a system of size $L = 53$, with temperature $T = 0.7T_{ons}$ and $p = 0$, where we have performed $N_{exp} = 500$ demagnetization experiments for each value of h. Error bars are smaller than symbol sizes.

Thus the system exhibits steady states for $g(m) = s(m)$. Fig. 8 shows $g(m)/N$ and $s(m)/N$ as measured in a system with size $L = 53$, temperature $T = 0.6T_{ons}$, $p = 0.005$ and $h = -0.1$, after averaging over 1000 different demagnetization experiments. These demagnetization experiments begin with all spins up (such state is metastable for the studied parameters) and finish once the negative magnetization stable state has been reached. There are three points where the curves $g(m)$ and $s(m)$ intersect one each other. Two of these intersection points appear in the positive magnetization region, and the third one appears in the negative magnetization branch. The points where $g(m) = s(m)$ signal steady states of the real system, whose magnetization can be deduced from the intersection abscissa. We denote these magnetization values as m_{-1}, m_0 and m_1, being m_{-1} the magnetization of the intersection point in the negative magnetization region, m_0 the magnetization of the intermediate intersection point, and m_1 the largest intersection point magnetization. In order to discern local stability, we study what happens if we slightly perturb the magnetization in these steady states. If we perturb for instance the steady state with the largest magnetization, m_1, in such a way that the final state has magnetization $m = m_1 + \delta m$, we can see that if $\delta m > 0$ then $g(m_1 + \delta m) > s(m_1 + \delta m)$, while $g(m_1 + \delta m) < s(m_1 + \delta m)$ if $\delta m < 0$. In both cases, as indicated by eq. (25), the system tends to counteract the perturbation, coming back to the stationary state. Hence the stationary state with the largest magnetization, m_1, is locally stable under small perturbations. The arrows in the inset of Fig. 8 represent the tendency of the system immediately after the perturbation. We find something analogous for the steady state with negative magnetization, m_{-1}, i.e. it is locally stable. Therefore the stationary state represented by m_1 signals the metastable state, while the stationary state m_{-1} signals the stable state in this case ($h < 0$). The steady state m_0 is unstable under small perturbations. This stationary solution signals the crossover point between the region where the stable phase tends to disappear ($m > m_0$), and the region where the stable phase tends to grow ($m < m_0$). In fact, this point defines the critical fluctuation needed in order to exit the metastable state. This critical fluctuation controls the demagnetization process. On the other hand, measuring $g(m)$ y $s(m)$ in particular experiments, extracting the stable and metastable state magnetizations, m_{-1} and m_1 respectively, and averaging such measures over many different experiments, we can obtain a measure of the average stable and metastable state magnetizations. This measure compares perfectly with the previously presented results (see Fig. 3 and 4, and complementary discussion).

It is clear from the previous discussion that if there is a magnetization interval inside the metastable region (in our case, the positive magnetization region) where $s(m) > g(m)$, that is, where the stable phase shrinkage rate is larger than its growth rate, then a metastable state will exists. On the other hand, when $|h| > h^*(T,p)$ the metastable state will not exist. In this case it is observed that the curve $s(m)$ does not intersect $g(m)$ in the positive magnetization region. Hence the existence or absence of intersection between $s(m)$ and $g(m)$ in the positive magnetization region (for $h < 0$) allows us to decide whether the system exhibits a metastable state or not. In the above discussion we have treated all states with the same magnetization as an single state. However, there are many different microscopic states in the system which are compatible with a fixed magnetization. These states may exhibit very different properties. In particular, the rates $g(m)$ y $s(m)$ depend not only on magnetization, but on the population of all the spin classes. Thus, for a fixed set of parameters T, p and $h < 0$, we can have experiments where $s(m)$ and $g(m)$ intersect one each other in the positive magnetization region, and for the same parameters we can observe other experiments where they do not intersect. Therefore, instead of speaking about the existence or absence of a metastable state, we must speak about the *probability* of existence of a metastable state. In this way we can define a method to measure the intrinsic coercive field $h^*(T,p)$ in Monte Carlo simulations. For a fixed set of parameters T, p and $h < 0$, we perform N_{exp} different demagnetization experiments, starting from a state with all spins up. We measure on each experiment the stable phase growth and shrinkage rates, $g(m)$ and $s(m)$ respectively, as a function of magnetization. If n_{met} of those N_{exp} experiments are such that $g(m)$ and $s(m)$ intersect one each other in the positive magnetization region, we can define the probability of existence of a metastable state as $n_{met}(T,p,h)/N_{exp}$. If we repeat such process for fixed values of temperature T and nonequilibrium perturbation p, varying the

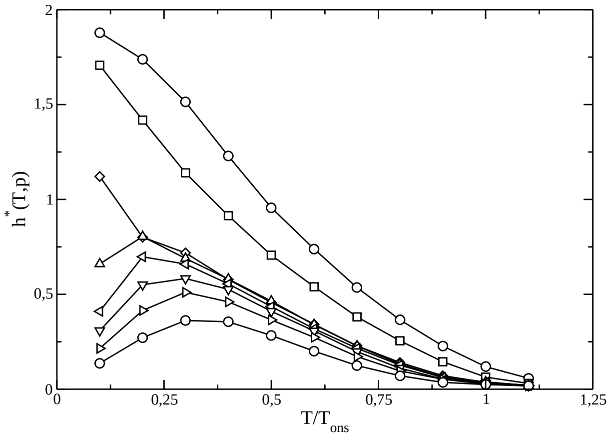

FIG. 10: Monte Carlo results for the intrinsic coercive field, $h^*(T,p)$, as a function of temperature for different values of p. In particular, from top to bottom, $p = 0$, 0.01, 0.03, 0.0305, 0.0320, 0.0350, 0.04 and 0.05. Notice the change of asymptotic behavior in the low temperature limit for $p \in (0.03, 0.0305)$. This figure is to be compared with Fig. 7.

magnetic field in a wide interval, we obtain the results shown in Fig. 9. Here we observe that the metastable state existence probability abruptly changes from +1 to 0 in a narrow magnetic field interval. Therefore we define in this case the intrinsic coercive field, $h^*(T,p)$, for fixed T and p, as the magnetic field strength for which $n_{met}(T,p,h^*)/N_{exp} = 0.5$.

Fig. 10 shows $h^*(T,p)$, as measured from Monte Carlo simulations using the above explained method, as a function of temperature for a system size $L = 53$, and for varying values of p. Comparing this figure with Fig. 7, we observe that Monte Carlo results confirm both qualitatively and quantitatively[52] the theoretical predictions based on Pair Approximation. In this way we observe that the low temperature asymptotic behavior of $h^*(T,p)$ depends on the nonequilibrium parameter p. There is a critical value π_c for p which separates both asymptotic behaviors. We estimate from Monte Carlo simulations $\pi_c^{MC} \approx 0.03025$ (see Fig. 10). This critical value has to be compared with the result derived from Pair Approximation, $\pi_c^{pair} \approx 0.0315$. Hence we confirm that the system exhibits, as we discussed above, a non-linear cooperative phenomenon between the thermal noise (T), and the non-thermal noise (p), for $p > \pi_c$, in such a way that there are no metastable states for low and high temperatures, but there is an intermediate temperature region where metastable states emerge due to the non-linear coupling between both noises.

ME QUEDO AQUI. LO QUE SIGUE HAY QUE MEJORARLO/REVISARLO

DISCUSSION

INTRODUCIR AQUI DISCUSION SOBRE LA PRESENCIA DE RUIDO MULTIPLICATIVO. LA DISCUSION SE DIVIDIRIA EN VARIAS PARTES:

1. EL EFECTO FISICO DE P SE PUEDE ENTENDER EN TERMINOS DE UNA TEMPERATURA EFECTIVA.

2. LA TEMPERATURA EFECTIVA ASOCIADA A CIERTO SPIN DEPENDE DEL ORDEN LOCAL AL QUE ESTA SUJETO DICHO ESPIN.

3. DE ESTA MANERA, PODEMOS ESCRIBIR DE FORMA HEURISTICA UNA ECUACION DE CAMPOS DE TIPO LANGEVIN PARA LA EVOLUCION DE NUESTRO SISTEMA. ESTA ECUACION SERIA MUY PARECIDA A LA TIPICA FI4 CON CAMPO MAGNETICO EXTERNO, PERO EL TERMINO DE RUIDO GAUSSIANO, EN VEZ DE IR MULTIPLICADO POR UNA AMPLITUD 1, IRIA MULTIPLICADO POR UNA AMPLITUD [1+mu*FI(x,t)**2]/2. DE ESTA MANERA, A MAYOR ORDEN LOCAL (MAYOR VALOR ABSOLUTO DE LA MAGNETIZACION LOCAL FI(x,t)), MAS GRANDES SERAN LAS FLUCTUACIONES DEL PARAMETRO DE ORDEN LOCAL, LO QUE FENOMENOLOGICAMENTE CONCUERDA CON LO QUE SE OBSERVA EN EL MODELO MICROSCOPICO. EL PARAMETRO mu DE LA TEORIA DE CAMPOS ESTA RELACIONADO CON EL PARAMETRO P DEL MODELO MICROSCOPICO, DE MANERA QUE CUANDO P=0 (EQUILIBRIO A TEMPERATURA T), DEBEMOS TENER QUE mu=0, MIENTRAS QUE CUANDO P=1 (EQUILIBRIO A TEMPERATURA INFINITA) mu DEBE TOMAR UN VALOR MUY GRANDE, DE MANERA QUE LA ECUACION DE LANGEVIN PARA EL CAMPO FI(x,t) SEA BASICAMENTE ALEATORIA (DE TIPO MOVIMIENTO BROWNIANO ??).

4. LA INTRODUCCION DE LA AMPLITUD [1+mu*FI(x,t)**2]/2 DA LUGAR A RUIDO MULTIPLICATIVO EN EL SISTEMA, LO QUE YA ES INTERESANTE DE POR SI. ESTO PUEDE DAR LUGAR A OTRO TRABAJO COMPLETO, SI SOMOS CAPACES DE ANALIZAR TEORICAMENTE (Y/O NUMERICAMENTE) LA NUCLEACION EN ESTA TEORIA DE CAMPOS (VER REVIEW DE NUCLEACION EN TEORIA DE CAMPOS DE GUNTON, SAN MIGUEL Y SAHNI). SI TODO

VA BIEN, LOS RESULTADOS DEBEN CONCORDAR CON LOS RESULTADOS CONTRA-INTUITIVOS QUE HEMOS OBSERVADO EN EL MODELO MICROSCOPICO, Y DARIA MUCHA SOLIDEZ A TODO NUESTRO ANALISIS.

5. OTRO PUNTO A FAVOR DE LA TEORIA DE CAMPOS MODIFICADA ES QUE SIGUE PERTENECIENDO A LA CLASE DE UNIVERSALIDAD DE ISING, TAL Y COMO SUCEDE PARA EL MODELO MICROSCOPICO.

CONCLUSION

AQUI HAY QUE RECORTAR MUCHO BLA, BLA. HAY QUE METER UNA PEQUENIA PARTE EXPLICANDO LOS PROBLEMAS QUE HAY PARA DESCRIBIR LA DINAMICA DE LA TRANSICION METAESTABLE-ESTABLE EN CAMPO MEDIO: AUSENCIA DE FLUCTUACIONES E HIPOTESIS DE HOMOGENEIDAD. EL PRIMER PROBLEMA SE PUEDE SOLUCIONAR INTRODUCIENDO FLUCTUACIONES DE MANERA NATURAL EN LA TEORIA DE CAMPO MEDIO: MEAN FIELD STOCHASTIC DYNAMICS (CITAR TESIS AQUI). sIN EMBARGO, EL PROBLEMA DE LA HOMOGENEIDAD ES INSALVABLE EN CAMPO MEDIO, Y NOS HACE CONCLUIR LA NECESIDAD DE UNA TEORIA DE LA NUCLEACION DE NO EQUILIBRIO PARA ESTE SISTEMA.

POR ULTIMO, SE RESUME TAMBIEN NUESTRA HIPOTESIS PLAUSIBLE SOBRE LA PRESENCIA DE RUIDO MULTIPLICATIVO EN ESTE SISTEMA, APUNTANDO SU RESPONSABILIDAD EN EL EFECTO COOPERATIVO NO LINEAL OBSERVADO ENTRE EL RUIDO DE ORIGEN TERMICO Y EL DE ORIGEN NO TERMICO. SE FINALIZA DICIENDO QUE ESTE ULTIMO EXTREMO SERA ESTUDIADO EN UN TRABAJO PROXIMO.

In this paper we have studied the static properties of metastable states in a nonequilibrium ferromagnetic model using a first order dynamic mean field approximation.

In particular, we have applied the so-called Pair Approximation[27, 28], a dynamic analogous of the equilibrium Bethe-Peierls Approximation, to the problem of metastability in our lattice spin system. This theory is based on a mean field approximation for the master equation governing the system dynamics, once this stochastic equation is reduced to local observables. The approximation is developed using three fundamental hypothesis. In a first step, it neglects all fluctuations in the system, so in this approach we only study the *average* behavior of local observables. On the other hand, this theory also neglects long range correlations. In particular, we only have into account nearest neighbor correlations. The last hypothesis assumes that the system is homogeneous, which implies that all points in the lattice behave in the same way, independently from their positions.

Taking into account these hypothesis, and taking as starting point the master equation, we obtain two coupled non-linear differential equations for the dynamics of x, the probability of finding an up spin in the system, and z, the probability of finding a $(+,+)$ nearest neighbors pair in the system. We obtain numerically the locally stable steady solutions of this set of differential equations, both for zero magnetic field and $h<0$. For $h=0$ we obtain theoretical predictions for the stationary state magnetization as a function of temperature for different values of p. These predictions perfectly compare with Monte Carlo results in the low and intermediate temperature regime, although some differences between theory and simulation appear for temperatures near to the critical one, $T_c(p)$, since for these temperatures long range correlations become important. As the value of the nonequilibrium perturbation p is increased, the stationary state magnetization decreases in magnitude for $h=0$ for a fixed temperature, although the qualitative shape of curves $m_s^{(\pm)}(T,p)$ is similar to those of the equilibrium system. The critical temperature $T_c(p)$ signals a second order phase transition in the nonequilibrium systems between a disordered phase for high temperatures and an ordered phase for low temperatures. Applying the marginal stability condition to the dynamic equations, we are able to extract the phase diagram in first order mean field approximation for the nonequilibrium model. The phase diagram yields the critical temperature $T_c(p)$ as a function of the nonequilibrium parameter p. The ordered phase disappears for all temperatures when $p>p_c$, where $p_c=\frac{5}{32}$ in this approximation. Finally, for the locally stable steady magnetization for $h<0$ we obtain qualitatively similar results as compared with the $h=0$ case, although now the up-down symmetry which held for $h=0$ breaks up. The comparison of predicted curves with Monte Carlo results for both magnetization branches for $h<0$ is also excellent.

On the other hand, the system exhibits hysteresis due to the existence of metastable states. This implies that the system keeps memory of the past evolution history. In particular, using mean field approximation we calculate the intrinsic coercive field $h^*(T,p)$, defined in this case as the magnetic field for which the metastable state becomes unstable. We observe that $h^*(T,p)$ shows two different kinds of asymptotic behaviors in the low temperature limit, which depend on the value of p. There is a critical value for p, $\pi_c \approx 0.0315$, which separates both behaviors. For $p<\pi_c$ the intrinsic coercive field $h^*(T,p)$ increases in magnitude as temperature decreases, in the same way that in equilibrium systems. However, for $p>\pi_c$ we predict that the intrinsic coercive field converges towards zero in the limit $T\to 0$, showing a maximum in magnitude for certain intermediate temperature. This involves the existence of a *non-linear cooperative phenomenon* between the thermal noise (parameterized by T) and the non-thermal noise (parameterized by p): although both noise sources independently add disorder to the system, which implies the attenuation, or even destruction of existing metastable states, the combination of both noises parameterized in the microscopic dynamics does not always involves a larger disorder, giving rise to parameter space regions where there are no metastable states for low and high temperatures, but metastable states appear for intermediate temperatures. This theoretical prediction based on the mean field approximation is fully confirmed via Monte Carlo simulations.

Finally, apart from the mean field investigations on the static properties of both stable and metastable states in the system, summarized in previous paragraphs, we have also attempted a description of the dynamics of the metastable-stable transition using Pair Approximation. However, one of the basic hypothesis in this approximation, namely the hypothesis of suppression of fluctuations, impedes any realistic description of this dynamic process using Pair Approximation. The reason underlies in that fluctuations constitute the basic mechanism which gives rise to the metastable-stable transition. Therefore, in order to describe the metastable state demagnetization process, we relax the above hypothesis, including fluctuations in the dynamic mean field theory. This can be done in a natural way using the concepts of stable phase growth and shrinkage rates, observables which are defined in a simple manner in our approximation, and the philosophy underlying MCAMC algorithms (see Appendix **??**). In this way we write a *mean field stochastic dynamics*, which includes fluctuations in a natural way. From this extended theory we predict the dynamic (and static) properties of the system. However, while the static results obtained from the mean field stochastic dynamics are equivalent to those obtained in Pair Approximation and reproduce the measured properties in Monte Carlo simulations, the results on the dynamics of the metastable-stable transition are remarkably different from those obtained in simulations. This discrepancy is due to the failure of another basic hypothesis of mean field approximation: the homogeneity hypothesis. This hypothesis implies that the exit from the metastable state in the mean field stochastic dynamics approximation is produced by *coherent* fluctuations of all spins in the system, which is energetically punished. The metastable demagnetization process in the real system is, on the other hand, an highly inhomogeneous process, where one or several stable phase droplets nucleate in the metastable bulk, since these compact structures minimize the system free energy for a fixed magnetization. Thus, in order to understand the dynamics of the metastable-stable transition we must therefore write an inhomogeneous theory where the interface plays a very important role. This inhomogeneous theory, based on the droplet picture, will be developed in Chapter **??**. However, in order to write such theory for the nonequilibrium system, we must first understand the interfacial properties in the model, since they will play a fundamental role in the droplet nucleation process. We study this problem in the next chapter.

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