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Physica A 296 (2001) 364–374

PHYSICA A

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Entropic contributions in Langevin equations for anisotropic driven systems

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Received 9 October 2000

Abstract

We report on analytical results for a series of anisotropic driven systems in the context of a recently proposed Langevin equation approach. In a recent paper (P.L. Garrido et al., Phys. Rev. E 61 (2000) R4683) we have pointed out that entropic contributions, over-looked in previous works, are crucial in order to obtain suitable Langevin descriptions of driven lattice gases. Here, we present a more detailed derivation and justification of the entropic term for the standard driven lattice gas, and also we extend the improved approach to other anisotropic driven systems, namely: (i) the randomly driven lattice gas, (ii) the two-temperature model and, (iii) the bi-layer lattice gas. It is shown that the two-temperature model and the lattice gas driven either by a random field or by an uniform infinite one are members of the same universality class. When the drive is uniform and finite the 'standard' theory is recovered. A Langevin equation describing the phenomenology of the bi-layer lattice gas is also presented. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 64.60.Cn

Keywords: Order-disorder transformations; Statistical mechanics of model systems

1. Introduction

When a system is driven out of thermal equilibrium its properties may change significantly. An illustrative example of this is the *driven lattice gas* (DLG) model

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[1] Supported by the European Network Contract ERBFMRXCT980183.

[2] Supported by the Ministerio de Educación under project DGESEIC, PB97-0842.

PII: S0378-4371(01)00043-7

[1–4], which has become a cornerstone for the study of nonequilibrium behavior, and is currently in the forefront of research activity in nonequilibrum statistical mechanics. Some time ago, motivated by discrepancies between Monte Carlo results [4–7] and field theoretic predictions [17,8] the general validity of the standard field theory for the DLG was questioned, and an alternative Langevin equation, put forward [9,10]. This alternative approach is certainly valuable *per se* because it represents a more detailed connection between microscopic models and their mesoscopic (Langevin) descriptions than previous more heuristic approaches.

However, some technical deficiencies in the emerging Langevin equation for the DLG under infinitely fast drive were pointed out by Caracciolo et al. [11] and also by Schmittmann et al. [12]. In particular, these authors identified the presence of unphysical generic infrared singularities in the alternative Langevin equation [11,12]: a clear indication of some flaw in our equation building. Nevertheless, as we have shown in our most recent article [13], the aforementioned deficiencies can be overcome once entropic contributions (not carefully separated from energetic ones in our first works) are properly handled. This improved approach led us to the following conclusions: (1) the driven lattice gas under finite driving field is represented by the standard model or Langevin equation, as conjectured some time ago by Cardy and Leung [17] (see also [8]); (2) in the case of infinite drive, we find a different Langevin equation with no current term. In a nutshell, this is due to the fact that for infinitely large drive there is no explicit dependence of the transition rates on the value of the density field, i.e. in the microscopic model if an attempted jump is feasible then it is performed regardless of the energetics of the neighboring lattice configuration, and consequently at a mesoscopic level the dependence of the transition rates on Ising energetics is altered. In this paper, we shall explicitely show how the abovementioned entropic corrections come about, and what is their role in the standard DLG.

Closely related to the DLG, there exists a whole repertoire of variants of it that display similar counterintuitive features. It is natural to investigate them so as to deepen our understanding of driven systems, and elucidate the points relevant in characterizing their collective behavior. In a recent paper [10], this issue was tackled as an application of, and a test for, the formalism previously developed for the DLG. Besides the DLG itself, three anisotropic driven systems were studied in [10]: the *randomly driven lattice gas*, *the two-temperature* model and the *bi-layer driven lattice gas*. A set of Langevin equations describing the phase transitions in these systems was provided [10]. However, all the calculations in [10] where affected by the previously mentioned problem, i.e., entropic contributions were not properly separated from energetic ones. It is our purpose here to heal that problem and see how those Langevin equations and the associated physical behavior are modified once entropic terms are introduced, and how these eventual modifications affect the emerging physical behavior.

This paper is organized as follows: in Section 2 we provide a detailed derivation of the Langevin equation for the DLG in which energetic and entropic contributions are properly distinguished. In the subsequent sections, we review the results reported in

[10] for the other three aforementioned anisotropic driven systems, and introduce the pertinent entropic corrections. Finally, our main conclusions are summarized.

2. The driven lattice gas

The DLG is a half-filled lattice gas coupled to a thermal bath at temperature T in which nearest-neighbor particle-hole exchanges are stochastically performed. The hopping rates are controlled by the Ising Hamiltonian and an external uniform driving field $\mathbf{E}$ pointing along one of the principal axis of the lattice. Periodic boundary conditions are imposed. The field biases the rates, favoring jumps along its direction, suppressing jumps against it, and leaving unaffected those in transverse directions. At some temperature T the DLG undergoes a nonequilibrium phase transition from a disordered state to an ordered one with a stripe-shaped particle-rich region parallel to the drive. This transition has been thoroughly studied in recent years [8,4]. Here, we shall not dwell any further on the system properties and refer the reader to [3,4] for detailed comprehensive reviews.

Let us now review our derivation of a mesoscopic Langevin equation for the DLG [9,10]. We define a coarse-grained excess particle density field $\phi(\mathbf{r},t)$ as the deviation of the actual density from its uniform average. Now, with each field configuration, $C=\{\phi\}$, we associate a statistical weight, $P_t(C)$, which evolves accordingly to the following continuous master equation [10]:

$$\partial(t)P_t(C)=\sum_a \int_R \mathrm{d}\eta\, f(\eta)\int \mathrm{d}\mathbf{r}$$
$$\times\{W[C^{\eta\mathbf{r}a}\to C]P_t(C^{\eta\mathbf{r}a})-W[C\to C^{\eta\mathbf{r}a}]P_t(C)\}\,. \tag{1}$$

Here, $W[C\to C']$ stands for the transition rates, $C^{\eta\mathbf{r}a}=\{\phi(\mathbf{x})+\eta\varepsilon\nabla_{\mathbf{x}_a}\delta(\mathbf{x}-\mathbf{r})\}$, $f(\eta)$ is an even function of η, and ε^{-1} is the volume over which the original microscopic occupation variables were averaged out. That is, Eq. (1) represents a process in which a "mass" $\varepsilon\eta$ is exchanged with the infinitesimal neighbor of $\mathbf{r}$ in the $\mathbf{a}$ direction. The "amount of mass", η, attempted to be displaced is distributed with a probability function $f(\eta)$, and the gradient in the definition of $C^{\eta\mathbf{r}a}$ ensures mass conservation. As it is usually done, the transition rates are taken to be a function of the free energy difference between the configurations plus a term due to the effect of the driving field, namely:

$$W[C\to C']=D(F(C')-F(C)+H_E(C\to C'))\,, \tag{2}$$

where F is the usual Ginzburg–Landau free energy:

$$F=\varepsilon^{-1}\int \mathrm{d}\mathbf{r}\left\{\frac{1}{2}(\nabla\phi)^2+\frac{\tau}{2}\phi^2+\frac{g}{4!}\phi^4\right\} \tag{3}$$

and

$$H_E(C\to C')=\eta\mathbf{a}\cdot\mathbf{E}(1-\phi(\mathbf{r})^2)+O(\varepsilon)\,. \tag{4}$$

The latter is a current term directed along the direction of $\mathbf{E}$ that accounts for the local variation of energy due to the driving field [3,17]. D is any function satisfying the detailed balance constraint, $D(-x) = e^x D(x) \geqslant 0$, which ensures that in the limiting case $E = 0$ the stationary distribution is the equilibrium one, $P_{st} \propto \exp(-F)$.

So far we have followed the derivation in our first papers [9,10]. Now we show where this reasoning fails and how it can be corrected.

For Eq. (2) in the limit of infinitely large driving forces, and jumps in the field direction, contributions from free energy variations are erased; i.e. allowed exchanges are performed with probability equal to 1 regardless of their surrounding configuration. One should recall that the coarse-grained free energy, F, comprises both entropic and energetic contributions.

The described situation is at odds with the microscopics of the DLG, where it is only the dependence on the Ising Hamiltonian that is washed away in the $E \to \infty$ limit. To see how to incorporate this key feature into our mesoscopic approach we consider the partition function of the microscopic equilibrium lattice gas model, i.e., the $E = 0$ limit, Z. As it is well known, a Gaussian transformation of Z together with a naive power counting analysis leads to a derivation of the Ginzburg–Landau free energy, F, at a mesoscopic level [14,15]. In particular, at the intermediate stage before arguments of relevance are applied, the structure of F consists of a bilinear-in-ϕ form, $1/T \sum \phi(\mathbf{k})\phi(-\mathbf{k})$ plus a term In cosh ϕ [14]. It is this different dependence on temperature that enables us to identify the entropic contribution to F as the one coming from the expansion of the In cosh ϕ:

$$S(C) = \varepsilon^{-1} \int d\mathbf{r} \left\{ \frac{\bar{\tau}}{2}\phi^2 + \frac{g}{4!}\phi^4 \right\} , \tag{5}$$

while the energetic term (coming from the gradient expansion of the bilinear form) reads:

$$H(C) = \varepsilon^{-1} \int d\mathbf{r} \left\{ \frac{1}{2}(\nabla\phi)^2 + \frac{\tau}{2}\phi^2 \right\} . \tag{6}$$

Accordingly, we rewrite the transition rates in the form:

$$W[C \to C'] = D(\Delta S(C))D(\Delta H(C) + H_E) , \tag{7}$$

$\Delta X(C)$ being equal to $X(C') - X(C)$. Note that, in doing so, the increment of energy from the drive enters the dynamics through $\Delta H + H_E$ and does not compete additively with ΔS. Moreover, using Eq. (7), when $E = 0$ the detailed balance condition on the rates is preserved, ensuing that in the zero field limit we recover the Ginzburg–Landau equilibrium distribution [25].

Next, a Kramers–Moyal [16] expansion of the Master equation yields the following Fokker–Planck equation (the detailed calculus can be found in [10]):

$$\partial_t P_t = \sum_a \int d\mathbf{r} \left(\nabla_{\mathbf{r}_a} \frac{\delta}{\delta\phi(\mathbf{r})} \right) \left\{ \varepsilon \bar{h}(\lambda_a^S, \lambda_a^H + \lambda_a^E) P_t \right.$$

$$\left. + \frac{\varepsilon^2}{2} \bar{e}(\lambda_a^S, \lambda_a^H + \lambda_a^E) \left(\nabla_{\mathbf{r}_a} \frac{\delta}{\delta\phi(\mathbf{r})} \right) P_t(C) \right\} , \tag{8}$$

where

$$\lambda_a^E \equiv \mathbf{a} \cdot \mathbf{E}(1 - \phi^2) ,$$

$$\lambda_a^X \equiv - \left(\nabla_{\mathbf{r}_a} \frac{\delta}{\delta\phi} \right) X ,$$

$$\bar{h}(x, y) \equiv \int_R d\eta \, f(\eta) \eta D(\eta x) D(\eta y) ,$$

$$\bar{e}(x, y) \equiv \int_R d\eta \, f(\eta) \eta^2 D(\eta x) D(\eta y) . \tag{9}$$

As the combination $\bar{h}(0,x)$ and $\bar{e}(0,x)$ will occur often, we introduce the symbols $h(x) = \bar{h}(0,x), e(x) = \bar{e}(0,x)$ which, besides, are closer to the notation of [6,7].

The equivalent Langevin equation reads (Ito sense) [16]:

$$\partial_t \phi(\mathbf{r}, t) = \sum_a \nabla_{\mathbf{r}_a} [\bar{h}(\lambda_a^S, \lambda_a^H + \lambda_a^E) + \bar{e}(\lambda_a^S, \lambda_a^H + \lambda_a^E)^{1/2} \zeta_a(\mathbf{r}, t)] , \tag{10}$$

with $\zeta_a(\mathbf{r}, t)$ being a Gaussian white noise, i.e., $\langle \zeta_a(\mathbf{r}, t) \rangle = 0$ and $\langle \zeta_a(\mathbf{r}, t) \zeta_{a'}(\mathbf{r}', t') \rangle = \delta_{a,a'} \delta(t - t') \delta(\mathbf{r} - \mathbf{r}')$. Time has been rescaled by a factor ε, and finally, ε has been fixed to 1. Eq. (10) is a slightly modified version of our previously proposed Langevin equation for the DLG (Eq. (7) of [9] or Eq. (15) in [10]) and, as in its older version, its dependence on microscopic details is apparent and contains the basic symmetries of the DLG.

We now focus on the critical region and discard irrelevant terms in the renormalization group sense by naive power counting. We perform the following scale transformations: $t \to \mu^{-z} t$, $r_\perp \to \mu^{-1} r_\perp$, $r_{||} \to \mu^{-\sigma} r_{||}$, and $\phi \to \mu^{\delta} \phi$, where $||$ stands for the direction parallel to the driving field $\mathbf{E}$, and $\perp$ for those perpendicular to it. The noise scales as $\zeta_a \to \mu^{(z+d+\sigma-1)/2} \zeta_a$. Next, we expand the Langevin equation around $\mu = 0$ while keeping only the leading terms. One realizes that the term $\bar{\tau} e(E)/2 \nabla_{||}^2 \phi$, absent in [9,10], is the only modification with respect to our old Langevin equation. Since this term does not vanish when $E = \infty$, $\nabla_\perp^4 \phi$ and $\nabla_{||}^2 \phi$ are the leading gradient terms of the theory, hence $\sigma = 2$.[3] The time scale, the transverse spatial interaction, and

[3] Due to the absence of parallel Laplacians, $\sigma = 1$ was used in [9]. In the present case that would be inconsistent with the presence of a unique tuning parameter.

the transverse noise are imposed to be invariant under scale transformations, implying $z = 4$ and $\delta = (\sigma + d - 3)/2$. This along with $\sigma = 2$, leads to:

$$\partial_t \phi(\mathbf{r}) = \frac{e_0}{2}\left[-\Delta_\perp^2 \phi + (\tau + \bar{\tau})\Delta_\perp \phi + \frac{g}{6}\Delta_\perp \phi^3\right]$$
$$+ \left(\bar{\tau}\frac{e(E)}{2} - \tau h'(E)\right)\nabla_{||}^2 \phi - Eh'(E)\nabla_{||}\phi^2$$
$$+ \sqrt{e_0}\sum\nolimits_\perp \nabla_\perp \zeta_\perp(\mathbf{r}, t) , \tag{11}$$

where h' is the first derivative of the function h and $e_0 = e(0)$. This is nothing but the equation postulated by Leung and Cardy [17] for the DLG, also known as *driven diffusive system* (DDS). The coarse-grained parameters introduced by Leung and Cardy appear here as specific functions of the Master equation parameters, and $E = |\mathbf{E}|$. Thus, we identify $Eh'(E)$ as the mesoscopic counterpart of the field $\mathbf{E}$, and $\bar{\tau} + \tau$ and $\bar{\tau}e(E)/2 - \tau h'(E)$ as the two effective temperatures associated with the transverse and longitudinal directions respectively [8]. Given that we know the detailed form of the coefficients in terms of the Master Equation parameters, we can explicitly verify that the limit $E \to \infty$ is peculiar, as for it the coefficient $Eh'(E)$ of the current term $(\nabla_{||}\phi^2)$ vanishes, and the relevant Langevin equation is:

$$\partial_t \phi(\mathbf{r}) = \frac{e_0}{2}\left[\Delta_\perp^2 \phi + (\bar{\tau} + \tau)\Delta_\perp \phi + \frac{\bar{\tau}}{2}\nabla_{||}^2 \phi + \frac{g}{6}\Delta_\perp \phi^3\right]$$
$$+ \sqrt{e_0}\sum_\perp \nabla_\perp \zeta_\perp(\mathbf{r}, t) . \tag{12}$$

Indeed, this equation was first proposed by [18] as the mesoscopic description of the random DLG, a DLG in which the driving field fluctuates accordingly to an even distribution of amplitudes (see Section 3 in this paper). It can be easily appreciated that the current term $Eh'(E)\nabla_{||}\phi^2$ is absent here. We conclude that the limit $E = \infty$ is a sort of multicritical point at which the current term coefficient vanishes due to the saturation of the transition rates in the Master equation. Consequently, we have named the universality class for the DLG in this limit *anisotropic driven system* (ADS) as *anisotropy* is the essential feature for the critical properties of the system. Remarkably, the value of the order parameter critical exponent that follows from a renormalization group analysis of Eq. (12) [18], $\beta \approx 0.33$ [19] agrees rather well with the estimations coming from Monte Carlo simulations of [4].

3. Randomly driven lattice gases

The *random driven lattice gas* (RDLG) is a DLG in which the driving force, although still pointing along a particular axis, has a random amplitude in time accordingly to an even distribution $p[E(\mathbf{x},t)]$ with δ-like correlations in space and time [18]. Hence, unlike the DLG, the $\phi \to -\phi$ and the $y \to -y$ (reflection in the direction of the field) symmetries are restored and no global current is present.

A Langevin equation for the RDLG can be easily derived by simply averaging out over the random E in (11). The term proportional to $Eh'(E)$ vanishes as a consequence of the symmetric $p[E]$ after integrating over η. This allows us to write:

$$\partial_t \phi(\mathbf{r}) = \frac{e_0}{2}\left[-\Delta_\perp^2 \phi + \tau_\perp \Delta_\perp \phi + \tau_{||} \nabla_{||}^2 \phi + \frac{g}{6}\Delta_\perp \phi^3\right]$$

$$+ \sqrt{e_0} \sum_\perp \nabla_\perp \zeta_\perp(\mathbf{r},t)\,, \qquad (13)$$

which coincides with Eq. (12), once the following identifications are done:

$$\tau_{||} = \frac{2}{e_0}\int \mathrm{d}E\, p[E]\left(\frac{\bar{\tau} e(E)}{2} - \tau h'(E)\right)\,,$$

$$\tau_\perp = \tau + \bar{\tau}\,. \qquad (14)$$

Therefore, we have recovered the mesoscopic picture for the RDLG postulated by Schmittmann and Zia in [18]. This makes an important difference with our previous analysis in [10]. There finite and infinite random drives yielded different Langevin equations. Here a single one is found since the infinite drive limit is not singular for the RDLG. Not surprisingly, it is the ADS universality class because no current term is present in this case.

4. Two-temperature model

The *two-temperature* (TT) model is an Ising half-filled lattice gas coupled to two thermal baths at different temperatures [20]. More specifically, particle-hole exchanges aligned with a particular direction are controlled by rates $D(\Delta H/T_{||})$, H being the Ising Hamiltonian. Otherwise they are coupled with a bath with temperature $T_\perp$, and controlled by rates $D(\Delta H/T_\perp)$. The TT model and the RDLG share the same set of symmetries so, although their microscopics details are quite different, it is expected that they both belong in the same universality class. In fact, Monte Carlo simulations for the TT model compare quite well with field theoretic predictions for the RDLG. For instance, $\beta \approx 0.33$ is measured by both procedures when $T_{||} = \infty$ [19,21].

In our context these similarities can be understood by simply nothing that the TT model corresponds to Eq. (1) without external drive and imposing two distinct τ mass terms in (6). Then, all results carry over without change leading to Eq. (12), thereby placing the TT model as another example of anisotropic driven system.

5. Bi-layer driven lattice gas

Let us consider a couple of DLG's copies stacked one on the top of the other without energetic coupling between the layers [22]. A given particle can hop to any of its five nearest neighbor sites (four in the same plane and one in the other one). Particle exchanges across layers are not affected by the drive and they are controlled by in-plane energetics alone. Finally, the overall particle density is fixed to $\frac{1}{2}$ to access the critical point [3,4]. As the temperature is decreased from a high enough value, the system first orders from a homogeneous state into one with high and low density stripes in both layers. This transition is much alike the one appearing in the standard DLG, i.e., each layer exhibits a phase segregated state with a linear liquid-vapor interface aligned with the applied field. Decreasing the temperature further, a second transition occurs to two homogeneously filled layers with different densities; particles accumulate in one of the two planes breaking spontaneously the symmetry between them. This transition is known to be Ising-like for any $E < E_{\rm c} \approx 2$, and becomes first order for values of E beyond the threshold field $E_{\rm c}$ [23,24]. We now proceed to investigate this issue further in the frame of our Langevin-equation-building approach.

We call $\phi_i(\mathbf{r},t)$ to the coarse grained excess density field in plane i, and $C = \{(\phi_1,\phi_2)\}$ denotes an arbitrary configuration. By $C_i^{\eta\mathbf{r}a}$ we term the configuration after an exchange of density $\varepsilon\eta$ is performed in the $\mathbf{a}$ direction with an infinitesimal neighbor of $\mathbf{r}$ in plane i, while $C^{\eta\mathbf{r}}$ stands for configurations obtained after exchanges between planes. More explicitely:

$$C_1^{\eta\mathbf{r}a} = \{\phi_1(\mathbf{x}) + \varepsilon\eta\nabla_{\mathbf{x}_a}\delta(\mathbf{x}-\mathbf{r}), \phi_2(\mathbf{x})\}\ ,$$

$$C_2^{\eta\mathbf{r}a} = \{\phi_1(\mathbf{x}), \phi_2(\mathbf{x}) + \varepsilon\eta\nabla_{\mathbf{x}_a}\delta(\mathbf{x}-\mathbf{r})\}\ ,$$

$$C^{\eta\mathbf{r}} = \{\phi_1(\mathbf{x}) + \varepsilon\eta\delta(\mathbf{x}-\mathbf{r}), \phi_2(\mathbf{x}) - \varepsilon\eta\delta(\mathbf{x}-\mathbf{r})\}\ . \tag{15}$$

Next, we write down the Master equation for these processes:

$$\begin{aligned}\partial_t P_t(C) = &\sum_a \sum_{i=1}^{2} \int \mathrm{d}\mathbf{r}\, d\eta\, f(\eta) \\ &\times\{W[C_i^{\eta\mathbf{r}a} \to C]P_t(C_i^{\eta\mathbf{r}a}) - W[C \to C_i^{\eta\mathbf{r}a}]P_t(C)\} \\ &+ \int \mathrm{d}\mathbf{r}\, d\eta\, f(\eta)\{W[C^{\eta\mathbf{r}} \to C]P_t(C^{\eta\mathbf{r}}) - W[C \to C^{\eta\mathbf{r}}]P_t(C)\}\ .\end{aligned} \tag{16}$$

If we now split the transition rates W as in Eq. (7), we obtain a Fokker–Planck equation which gathers the single layer contributions of (8) plus the intra-layer coupling term:

$$\int \mathrm{d}\mathbf{r} \left(\nabla_{12}\frac{\delta}{\delta\phi(\mathbf{r})}\right) \left\{\varepsilon\bar{h}(\lambda_{12}^s, \lambda_{12}^H)P_t + \frac{\varepsilon^2}{2}\bar{e}(\lambda_{12}^s, \lambda_{12}^H)\left(\nabla_{12}\frac{\delta}{\delta\phi(\mathbf{r})}\right)P_t\right\}\ . \tag{17}$$

With $\nabla_{12}\frac{\delta}{\delta\phi}$ we denote the operator $\delta/\delta\phi_1 - \delta/\delta\phi_2$ and λ_{12}^X stands for $(\nabla_{12}\frac{\delta}{\delta\phi})X(C)$. Lastly, we obtain the following Langevin equation:

$$\partial_t\phi_1 = \{\text{plane 1 part}\} + \bar{h}(\lambda_{12}^S, \lambda_{12}^H) + \bar{e}(\lambda_{12}^S, \lambda_{12}^H)^{1/2}\zeta\,,$$

$$\partial_t\phi_2 = \{\text{plane 2 part}\} - \bar{h}(\lambda_{12}^S, \lambda_{12}^H) - \bar{e}(\lambda_{12}^S, \lambda_{12}^H)^{1/2}\zeta\,, \tag{18}$$

where "plane i part" represents:

$$\sum_a \nabla_{\mathbf{r}a}[\bar{h}(\lambda_{a,i}^S, \lambda_{a,i}^H + \lambda_{a,i}^E) + \bar{e}(\lambda_{a,i}^S, \lambda_{a,i}^H + \lambda_{a,i}^E)\zeta_a^{(i)}]\,, \tag{19}$$

i.e., it is the same as in (10), the index i corresponds to either plane and ζ is a Gaussian white noise. [4]

Eq. (18) can be expressed in terms of the two new fields:

$$m(\mathbf{r}) \equiv (\phi_1 + \phi_2)/2, \quad \varphi(\mathbf{r}) \equiv (\phi_1 - \phi_2)/2\,. \tag{20}$$

We shall take $m(\mathbf{r})$ as the conserved order parameter for the first phase transition, and $\varphi(\mathbf{r})$ as the (non-conserved) order parameter for the second one. To show that all observed critical behaviors are contained in (18), we introduce an external momentum scale μ and perform the following scale transformations: $t \to \mu^{-z}t$, $r \to \mu^{-\sigma}r$, $\varphi \to \mu^{\delta}\varphi$, and $m \to \mu^{\gamma}m$. First, the exponents are fixed to $z = 4$, $\sigma = 2$, $\delta = d/2$ and $\gamma = (d-1)/2$, which are the same set of exponents used in the power counting of the DLG. The resulting Langevin equation for the field m then matches the Langevin equation for the DLG (11). As for the second transition, the one that occurs at a lower temperature, we use $z = 2$, $\sigma = 1$, $\delta = (d-2)/2$ and $\gamma = d/2$ for rescaling since good data collapse is obtained from isotropic finite-size scaling. Using the previous rescaling, the relevant part of the Langevin equation is naively consistent with the model A, the mesoscopic counterpart of the Ising model [25]. In fact, we found [10] that the problem in the vicinity of this second transition can be mapped exactly to the continuum description of the DLG with repulsive interactions [26]. Since that model has already been studied in [26], we now simply recall the conclusions drawn there: the evolution equation for the field φ results, as previously discussed, in a model A. The driving force does not couple directly to the ordering field and it influences the transition only through the non-ordering field $m(\mathbf{r})$. The native dimension of $\mathbf{E}$ turns out to be $(d-2)/2$ so it is naively irrelevant for the Gaussian fixed point until d is lower than two and its only effect consists in generating anisotropies. However, corrections to order $O(E^2)$ show that g and τ decrease to an amount that depends of E. Eventually, both of them may vanish simultaneously, a mechanism that would be liable for the tricritical point [26].

[4] Note that (18) was given incorrectly in [10] since there only the arguments of h appeared.

6. Conclusions

In this paper, we have revisited our recently proposed method to derive Langevin equations for anisotropic driven systems from the corresponding Master equations. The improved method overcomes some drawbacks of the original version of our approach. In particular, now entropic contributions are properly distinguished from energetic ones, and this heals a notorious flaw of the emerging Langevin equation for the DLG. Indeed, the resulting Langevin equation does not present generic infrared singularities. A scaling analysis of this new equation reveals two main conclusions: (1) for finite driving we recover the Langevin equation (sometimes called the "standard model") proposed sometime ago by Leung and Cardy [17,8] in order to represent the DLG. (2) In the limit of infinitely large driving field, due to a saturation effect in the transition rates, the final equation is different; it has no density-dependent current term, and leads to a different renormalization group fixed point. We call this last universality class ADS (anisotropic driven system).

We have also applied the general method to some other anisotropic DRIVEN systems. Namely the random driven lattice gas, the two-temperature model and the driven bi-layer lattice gas. This same task was tackled in [16], and here we revisit the corresponding derivation using the improved method. Our main results are: (1) for the randomly driven lattice gas we recover the ADS Langevin equation for both finite and infinite driving fields, (2) the two-temperature model is also describable by the ADS Langevin equation for either finite or infinite temperature values, (3) for the bi-layer driven lattice gas we have obtained a pair of coupled Langevin equations and shown that they describe properly the two transitions observed in simulations.

To sum up, we have explicitely shown how the results presented in [9,10] can be improved once entropic contributions are properly taken into account. As a conclusion we have developed a comprehensive method to build Langevin equations for a particularly important class of nonequilibrium systems. Hopefully, modifications of this procedure will permit us to study other interesting nonequilibrium situations from a theoretical point of view.

Acknowledgements

It is a pleasure to acknowledge J. Marro and S. Caracciolo and collaborators for useful discussions.

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