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NONEQUILIBRIUM MODEL OF NEURAL NETWORKS

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Abstract

We introduce a kinetic lattice model of neural networks whose main novel features are that it may present nonequilibrium steady states, that it generalizes certain previous proposals, and that it admits a relatively simple analytical treatment in some limiting conditions.

Introduction

The brain is nowadays idealized [1] as a large number of neurons interacting with each other through *synapsis*: when the sum of the signals a neuron receives through their *synapsis* is larger than a certain amount, the neuron itself fires a signal. Even though these elementary processes are simple, the global, observable or *macroscopic* behavior of a large set of neurons is exceptionally rich: they can learn, remind, compare, compute,..., usually much more efficiently than a large computer.

Getting complex macroscopic behavior from simple microscopic laws has been very successful in physics during the last few decades. In particular, statistical physics has shown how many relevant macroscopic phenomena, such as phase transitions, critical phenomena, hydrodynamics, etc., are a cooperative effect which is also present in oversimplified microscopic models, i.e. in mathematically well defined models which contain certain essential features which characterize different classes of natural systems. The philosophy and techniques of statistical physics also maybe applied to model the behavior of an ideal brain by considering the so called *neural networks*. Following some seminal work in the field [2] and previous studies of nonequilibrium disordered systems [3-6], we present here a lattice model with competing dynamics which may be relevant to the study of the neural network problem, e.g. in a sense it generalizes familiar model systems while presenting *nonequilibrium* steady states and instabilities.

Consider a d -dimensional lattice, Z^d . At each lattice site, \mathbf{x} , there is a variable that can take two values, $s_{\mathbf{x}} = \pm 1$, representing the two only possible states of the neuron in that site, i.e. firing or not a signal. The state of the system is defined by $\underline{s} = \{s_{\mathbf{x}}, \mathbf{x} \in Z^d\}$. Neurons interact through *synapsis* in such

a way that the contribution to the potential of the neuron at \mathbf{x} of a signal fired by the neuron at \mathbf{y} is J_{xy} ; the J 's are assumed to have either a genetic origin or to be acquired by a learning process. The total potential arriving to the neuron at \mathbf{x} is then defined:

$$V_{\mathbf{x}} = \sum_{\mathbf{y}} J_{xy} [s_{\mathbf{y}} + 1] \quad (1.1)$$

Assuming no other effects present, each neuron will fire a signal, acquiring the state $s_{\mathbf{x}} = +1$, when its potential is above a threshold value, $U_{\mathbf{x}}$; in other case it remains in its original state. That is, over each neuron there is a local field,

$$h_{\mathbf{x}}(\mathbf{s}) = V_{\mathbf{x}} - U_{\mathbf{x}} \quad (1.2)$$

such that always $h_{\mathbf{x}} s_{\mathbf{x}} > 0$.

Giving an initial neuron configuration, $\mathbf{s}(0)$, we define a dynamical mechanism to update it,

$$s_{\mathbf{x}}(t+1) = s_{\mathbf{x}}(t) \operatorname{sgn}[H_{\mathbf{x}}(\mathbf{s}(t)|J)] \quad \forall \mathbf{x} \in \mathbb{Z}^d \quad (1.3)$$

where

$$H_{\mathbf{x}}(\mathbf{s}|J) = h_{\mathbf{x}} s_{\mathbf{x}} = s_{\mathbf{x}} \left[\sum_{\mathbf{y}} J_{\mathbf{x},\mathbf{y}} s_{\mathbf{y}} + \sum_{\mathbf{y}} J_{\mathbf{x},\mathbf{y}} - U_{\mathbf{x}} \right] \quad (1.4)$$

In general, this dynamics will drive the system to an stationary state when $t \rightarrow \infty$. Following Hebb's hypothesis about learning [7], one locates memory in the synapses, that is, there is a one to one relation between an stored pattern of J 's and the stationary state associated to the dynamical process defined by equation (1.3). Then, we consider that we remind a concrete pattern when the state of the neurons are compatible with the stationary state associated with the pattern. Sometimes different patterns will have same states, implying there is not possible a process of pattern recognition and differentiation i.e. even though our perception has stored the patterns in the synapsis, we are not capable to remind them.

To complete and make more realistic the model it is necessary to notice that the dynamical process we built in equation (1.3) for the neurons evolution is fully deterministic and assumes fixed the value of synapsis. We know that real system evolutions involve some amount of uncertainty or noise which come from the interaction of the system with a thermal bath surrounding it. In our case we can think that the interaction of a neuron with some fluid and other cells around, or different chemical reactions taking place inside it, might produce, eventually, an spontaneous change of the neuron state independently of the potential energy produced by the other neurons. We assume that these processes don't follow any deterministic rule and, consequently, are of stochastic nature. Usually, the level of this noise is measured by a parameter T which is similar to the temperature for the usual equilibrium processes.

But not only the neurons might change spontaneously their state, the synapsis also can change their value over the time in an stochastic way. The process of storing the pattern in the synapsis is done in a finite time interval. In this interval the pattern might change, implying changes in the value of J 's. We assume that the synapsis doesn't memorize the last value of the pattern (i.e. J 's) but its average distribution during that time interval. Then, when the learning process has finished, the synapsis will retrieve patterns following the memorized distribution, $P(J)$. That is, a pattern will be characterized not by a fix configuration of J 's but by a fix distribution of J 's.

The relevant magnitude which take account of those stochastic processes is the probability to find the system in a neuron configuration \mathbf{s} and a synapsis configuration J at time t , $P_t(\mathbf{s}, J)$, which obeys a marcovian master equation:

$$\frac{\partial}{\partial t} P_t(\mathbf{s}, J) = \sum_{\mathbf{s}', J'} [P_t(\mathbf{s}', J') c(\mathbf{s}', J' \rightarrow \mathbf{s}, J) - P_t(\mathbf{s}, J) c(\mathbf{s}, J \rightarrow \mathbf{s}', J')] \quad (1.5)$$

where $c(\mathbf{s}, J \rightarrow \mathbf{s}', J')$ is the transition probability per unit time from the configuration \mathbf{s}, J to \mathbf{s}', J' . Given the rates, c , the dynamics of the process is completely defined and the solution of eq. (1.5) gives the behavior of $P_t(\mathbf{s}, J)$. Obviously all the dynamical properties we commented above have to be introduced in an adequate form in the rates. We assume in our model: 1) The stochastic change of the pattern does not depend on the neuron state configuration and, 2) the time scale we use is such that only one neuron or one synapsis can change its state in the unit time interval. Both assumptions are equivalent to write:

$$c(\mathbf{s}, J \rightarrow \mathbf{s}', J') = p c_1(\mathbf{s} \rightarrow \mathbf{s}' | J) \delta(J, J') + (1-p) c_2(J \rightarrow J') \delta(\mathbf{s}, \mathbf{s}') \quad (1.6)$$

where

$$c_1(\mathbf{s} \rightarrow \mathbf{s}' | J) = \sum_{\mathbf{y}} \left[\prod_{\mathbf{x} \neq \mathbf{y}} \delta(s_{\mathbf{x}}, s'_{\mathbf{x}}) \right] \delta(s_{\mathbf{y}}, -s'_{\mathbf{y}}) w_1(\mathbf{s}; \mathbf{y} | J) \quad (1.7)$$

$$c_2(J \rightarrow J') = \sum_{\langle \mathbf{x}, \mathbf{y} \rangle} \left[\prod_{\langle \mathbf{u}, \mathbf{v} \rangle \neq \langle \mathbf{x}, \mathbf{y} \rangle} \delta(J_{\mathbf{uv}}, J'_{\mathbf{xy}}) \right] (1 - \delta(J_{\mathbf{xy}}, J'_{\mathbf{xy}})) w_2(J_{\mathbf{xy}} \rightarrow J'_{\mathbf{xy}}) \quad (1.8)$$

and we have introduced the parameter $p \in [0, 1]$ which measures the probability of neuron processes with respect synapsis ones. The rates $w_1(\mathbf{s}; \mathbf{x} | J)$ and $w_2(J \rightarrow J')$ are the probabilities per unit time to do the change $s_{\mathbf{x}} \rightarrow -s_{\mathbf{x}}$, fixed J , and $J \rightarrow J'$, independently on \mathbf{s} , respectively. Introducing eqs. (1.6)-(1.8) in the master equation (1.5) we obtain:

$$\begin{aligned} \frac{\partial}{\partial t} P_t(\mathbf{s}, \mathcal{J}) = & p \sum_{\mathbf{y}} [w_1(\mathbf{s}^{\mathbf{y}}; \mathbf{y} | \mathcal{J}) P_t(\mathbf{s}^{\mathbf{y}}, \mathcal{J}) - w_1(\mathbf{s}; \mathbf{y} | \mathcal{J}) P_t(\mathbf{s}, \mathcal{J})] \\ & + (1-p) \sum_{\langle \mathbf{x}, \mathbf{y} \rangle} \sum_{J_{\mathbf{x}\mathbf{y}}^t} [w_2(J_{\mathbf{x}\mathbf{y}}^t - J_{\mathbf{x}\mathbf{y}}) P_t(\mathbf{s}, \mathcal{J}^{\mathbf{x}, \mathbf{y}}) - w_2(J_{\mathbf{x}\mathbf{y}} - J_{\mathbf{x}\mathbf{y}}^t) P_t(\mathbf{s}, \mathcal{J})] \end{aligned} \quad (1.9)$$

where

$$\begin{aligned} (\mathbf{s}^{\mathbf{y}})_x = & s_x, \quad \text{if } \mathbf{x} \neq \mathbf{y} \\ & -s_x, \quad \text{if } \mathbf{x} = \mathbf{y} \end{aligned} \quad (1.10)$$

$$\begin{aligned} (J^{\mathbf{x}, \mathbf{y}})_{uv} = & J_{uv}, \quad \text{if } \langle \mathbf{u}, \mathbf{v} \rangle \neq \langle \mathbf{x}, \mathbf{y} \rangle \\ & J_{uv}^t, \quad \text{if } \langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle \end{aligned}$$

and we choose

$$w_1(\mathbf{s}; \mathbf{x} | \mathcal{J}) = \Psi(2\beta H_{\mathbf{x}}(\mathbf{s} | \mathcal{J})) \quad (1.11)$$

where $\beta = (k_B T)^{-1}$ and Ψ is an arbitrary positive function having the so called "detailed balance" property, i.e. $\Psi(\mathbf{x}) = \Psi(-\mathbf{x}) e^{\mathbf{x}}$. Usual elections in the literature of these functions are: $\Psi(\mathbf{x}) = \min(1, e^{\mathbf{x}})$ (Metropolis), $\Psi(\mathbf{x}) = e^{-\mathbf{x}^2}$ (Van Beijeren-Schulmann) and $\Psi(\mathbf{x}) = 2(1+e^{\mathbf{x}})^{-1}$ (Kawasaki). Notice that the explicit form of w_2 will depend on the concrete type of stored pattern.

This model have two natural limits. When $p=1$ the evolution mechanism of J 's have probability zero to occurs, that is, the initial distribution of J 's remain fixed all over the time. In particular, when the interactions satisfy the symmetry property: $J_{\mathbf{x}\mathbf{y}} = J_{\mathbf{y}\mathbf{x}}$ and, because of the detailed balance property for the rates given by equation (1.11), the resulting stationary state is an equilibrium state characterized by the hamiltonian

$$H(\mathbf{s}) = - \sum_{\mathbf{x}} H_{\mathbf{x}}(\mathbf{s} | \mathcal{J}) = - \frac{1}{2} \sum_{\mathbf{x}, \mathbf{y}} J_{\mathbf{x}, \mathbf{y}} s_{\mathbf{x}} s_{\mathbf{y}} - \sum_{\mathbf{x}} h_{\mathbf{x}}^0 s_{\mathbf{x}}, \quad (1.12)$$

where

$$h_{\mathbf{x}}^0 = \frac{1}{2} \sum_{\mathbf{y}} J_{\mathbf{x}, \mathbf{y}} - U_{\mathbf{x}}. \quad (1.13)$$

Common realizations of this hamiltonian are: the well known Ising model when all the J 's are equal, i.e. $J_{\mathbf{x}\mathbf{y}} = J$, and the Edwards-Anderson spin-glass model [8] when the J 's are randomly distributed along the lattice with respect to a distribution $P_{BA}(\mathcal{J})$.

The other natural limit appears when $p \rightarrow 0$. In this case the synapsis evolution is much more probable than the neuron evolution in such a form that during the time interval between two neuron flips, the synapsis have evolved so fast that they are already in their stationary state. Then, we can distinguish a *microscopic* time scale, t , in which synapsis evolve from an initial distribution with neurons freezed, and a *macroscopic* time

scale, $\tau = pt$, $p \rightarrow 0$, $t \rightarrow \infty$ and τ finite, in which synapsis are already in their stationary state, $P_n(\mathcal{J})$, and neurons evolve. Then, in this limit, both time scales are separable and we can write different evolution equations for the synapsis and for the neurons,

$$\frac{\partial}{\partial t} P_t(\mathcal{J}) = \sum_{\langle x, y \rangle} \sum_{J'_{xy}} [w_2(J'_{xy} \rightarrow J_{xy}) P_t(\mathcal{J}^{xy}) - w_2(J_{xy} \rightarrow J'_{xy}) P_t(\mathcal{J})] \quad (1.14)$$

and

$$\frac{\partial}{\partial t} Q_t(\mathbf{s}) = \sum_{\mathbf{x}} [w_{eff}(\mathbf{s}^{\mathbf{x}}; \mathbf{x}) Q_t(\mathbf{s}^{\mathbf{x}}) - w_{eff}(\mathbf{s}; \mathbf{x}) Q_t(\mathbf{s})] \quad (1.15)$$

where

$$w_{eff}(\mathbf{s}; \mathbf{x}) = \int d\mathcal{J} P_{st}(\mathcal{J}) w_1(\mathbf{s}; \mathbf{x} | \mathcal{J}) \quad (1.16)$$

and $P_t(\mathcal{J}) \{Q_t(\mathbf{s})\}$ is the probability to find the configurations of synapsis {neurons} $\mathcal{J} \{\mathbf{s}\}$ at time $t \{\tau\}$. Notice that in the case $p=1$ the stationary state of the system may be an equilibrium one and then we can use the standard tools of the equilibrium statistical mechanics, for any other p values, in particular the limiting case $p \rightarrow 0$, the system has a nonequilibrium stationary state and actually there are not general theory as in equilibrium to study those systems, making the study and analysis of those cases much difficult.

To get some insight of our model we describe a mean field case. We assume that $J_{xy} = \xi_{xy}/N$, where N is the number of lattice sites and we introduce in equation (1.4) the mean field approximation by substituting the value of the neurons interacting with neuron \mathbf{x} by their mean value, i.e. $s_y \rightarrow \langle s_y \rangle$ in eq. (1.4), then we get

$$H_{\mathbf{x}}(\mathbf{s} | \xi) = s_{\mathbf{x}} \frac{1}{N} \sum_y \xi_{xy} \langle s_y \rangle \equiv s_{\mathbf{x}} m_{\mathbf{x}}(\xi) \quad (1.17)$$

where for simplicity we have taken $U_{\mathbf{x}} = \sum_y J_{xy}$. The first case we study is $p=1$, i.e. the synapsis configuration is frozen. Using the master equation (1.9), we realize that there is only one relevant dynamical equation

$$\frac{\partial}{\partial t} \langle s_{\mathbf{x}} \rangle_{\xi, t} = -2 \langle s_{\mathbf{x}} \Psi(2\beta s_{\mathbf{x}} m_{\mathbf{x}}(\xi)) \rangle_{\xi, t} \quad (1.18)$$

where $\langle \cdot \rangle_{\xi, t}$ indicates the average value of neurons with fixed configuration of synapsis at time t . The stationary solution of equation (1.18) is given by

$$\langle s_{\mathbf{x}} \rangle = \tanh(\beta m_{\mathbf{x}}(\xi)) \quad (1.19)$$

and the implicit equation

$$m_x(\xi) = \frac{1}{N} \sum_y \xi_{xy} \tanh(\beta m_y(\xi)) \quad (1.20)$$

In particular, if we choose

$$\xi_{xy} = \sum_{\alpha=1}^p \epsilon_x^\alpha \epsilon_y^\alpha \quad (1.21)$$

where $\epsilon_x^\alpha = \pm 1$, we obtain the neural network equations describing the model by Amit et al. [2], whose main feature is the existence of a second order phase transition at $\beta = \beta_c = 1$ from a disordered phase to a phase where $2p$ degenerate and thermodynamically stable states appear, each one associated to a learned pattern, $\{\epsilon^\alpha\}$. When $p = \alpha N$, the network can retrieve patterns, i.e. exists the phase transition described above, if $\alpha < \alpha_c = 0.14$, implying the existence of a maximum in the network storage capacity.

For the other limiting case, $p \rightarrow 0$, we use equations (1.15) and (1.16). Then, the stationary state for the neurons is given by

$$\langle s_x \rangle = \frac{\int d\xi P_{st}(\xi) [\Psi(-2\beta m_x(\xi)) - \Psi(2\beta m_x(\xi))]}{\int d\xi P_{st}(\xi) [\Psi(-2\beta m_x(\xi)) + \Psi(2\beta m_x(\xi))]} \quad (1.22)$$

where

$$m_x(\xi) = \frac{1}{N} \sum_y \xi_{xy} \langle s_y \rangle \quad (1.23)$$

and $P_{st}(\xi)$ is the stationary distribution of synapsis patterns. Let us study the particular case of a linear superposition of patterns, i.e.

$$P(\xi) = \sum_{\alpha=1}^p a_\alpha \delta(\xi - \eta^\alpha) \quad , \quad \sum_{\alpha=1}^p a_\alpha = 1 \quad (1.24)$$

Then, from equation (1.22) we get

$$m_x^v = \frac{1}{N} \sum_y \eta_{xy}^v \frac{\sum_{\alpha=1}^p a_\alpha [\Psi(-2\beta m_y^\alpha) - \Psi(2\beta m_y^\alpha)]}{\sum_{\alpha=1}^p a_\alpha [\Psi(-2\beta m_y^\alpha) + \Psi(2\beta m_y^\alpha)]} = \frac{1}{N} \sum_y \eta_{xy}^v \langle s_y \rangle \quad (1.25)$$

The system has a second order phase transition at inverse temperature, β_c , which is solution of the implicit equation

$$\det M = 0, \quad M_{xy} = \frac{\beta}{N} \sum_{\nu=1}^p a_{\nu} \eta_{xy}^{\nu} - \delta_{xy} \quad (1.26)$$

In particular when all synapsis of a pattern have the same strength, i.e. $\eta_{xy}^{\nu} = \eta^{\nu}$ (case 1), we get $\beta_c = [\sum_{\alpha} a_{\alpha} \eta^{\alpha}]^{-1}$; if the patterns are of Mattis type, i.e. $\eta_{xy}^{\nu} = \epsilon_x^{\nu} \epsilon_y^{\nu}$ with $\epsilon_x^{\nu} = \pm 1$ independent random variables (case 2), then $\beta_c = [\max_{\alpha} (a_{\alpha})]^{-1}$.

The zero temperature properties depend on how the rate $\Psi(x)$ behaves when $x \rightarrow -\infty$ that, in general, is of the form $\Psi(x) \sim f \cdot \exp(-\pi x)$. When $\pi=0$ and $T=0$ the equation (1.25) reduces to

$$m_x^{\nu} = \sum_{\alpha=1}^p a_{\alpha} \frac{1}{N} \sum_y \eta_{xy}^{\nu} \operatorname{sgn}(m_y^{\alpha}) \quad (1.27)$$

and, par example, in the case 1 we get

$$\langle s_x \rangle = \begin{cases} \pm \sum_{\alpha=1}^p a_{\alpha} \operatorname{sgn}(\eta^{\alpha}) & , \text{ if } \sum_{\alpha=1}^p a_{\alpha} \operatorname{sgn}(\eta^{\alpha}) > 0 \\ 0 & , \text{ otherwise} \end{cases}$$

and in the case 2

$$\overline{m^{\nu}} = \frac{1}{N} \sum_y \epsilon_y^{\nu} \langle s_y \rangle = \pm a_{\nu} \quad (1.29)$$

If $\pi \neq 0$ and $T=0$ the eq. (1.25) reads

$$m_x^{\nu} = \frac{1}{N} \sum_y \eta_{xy}^{\nu} \operatorname{sgn}(m_y^{\alpha}) \quad (1.30)$$

where $m_y^{\alpha} = \max_y m_y^{\nu}$. Then for case 1

$$\langle s_x \rangle = \begin{cases} \pm \max_y (\eta^{\nu}) & , \text{ if } \max_y (\eta^{\nu}) > 0 \\ 0 & , \text{ otherwise} \end{cases} \quad (1.31)$$

and for the case 2 we obtain

$$\overline{m^{\nu}} = \delta_{\alpha\nu}, \quad \alpha = 1, 2, \dots, p \quad (1.31)$$

Notice how in case 1 for both types of dynamics, the ground state of the system is degenerate but with only two states which are mixtures of all the patterns. Then, a distribution of J 's superposition of space homogeneous distributions, are useless to store patterns in the network. By contrast, in case 2 the ground state is p -degenerate. When $\pi=0$ we reproduce an original pattern ν with an error $1-a_{\nu}$ and, par example, if the patterns have the same probability to appear, i.e. $a_{\alpha} = 1/p$, then the error reproducing a pattern at zero temperature is $(p-1)/p$ and if p

goes to infinity the error is one. That is, the network can store an arbitrary large number of patterns but with large errors to reproduce them. In contrast, when $\eta \neq 0$, the network can store and retrieve any number of patterns without error, being this, the most relevant feature of the network.

Also remark the relevant role that neuron dynamics plays in the macroscopic system behavior to compare with the usual equilibrium systems in which dynamics is irrelevant. There is work in progress to study the attraction domains of these solutions, their stability properties and the influence of different patterns and dynamics on the system behavior.

We have seen in the above mean field model of a neural network, how the distribution of J 's, the explicit form of the rates and the value of the parameter p determine the macroscopic behavior of the network. In future work, the study of the learning process, the influence of the p parameter in the retrieval of the memories and the use of models with a finite range of interaction between neurons, opens a new way in the understanding of the neural networks systems.

References:

- [1] S. Ramón y Cajal, Histologie du Système Nerveux, CSIC, Madrid.
- [2] D.J. Amit, H. Gutfreund and H. Sompolinsky, *Phys. Rev. A* 32, 1007-1018 (1985); *Ann. of Phys. (N.Y.)* 173, 30-67 (1987).
- [3] P.L. Garrido and J. Marro, *Phys. Rev. Lett* 62, 1929 (1989).
- [4] A.I. López-Lacomba, P.L. Garrido and J. Marro, *J. Phys. A* 23, 3809 (1990).
- [5] P.L. Garrido and J. Marro : "A nonequilibrium version of the spin-glass problem", *Europhysics Letters* in press.
- [6] A.I. López-Lacomba, J. Marro and P.L. Garrido: "Magnetic system under a fluctuating field", preprint.
- [7] D.O. Hebb, The Organization of behavior, Wiley, New York, 1949.
- [8] S.F. Edwards and P.W. Anderson, *J.Phys. F* 5, 965 (1975).