Phase transition in quantum annealing of hard problems detected by fidelity susceptibility

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We numerically analyze the quantum annealing of an NP-hard problem, namely the maximum independent set problem in the hard regime. The system is expressed by a Hamiltonian of the form

$$\hat{H}(\lambda) = (1-\lambda)\left(\sum_{\langle i,j\rangle} J_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z + \sum_i h_i\hat{\sigma}_i^z\right) + \lambda\left(\sum_i \hat{\sigma}_i^x\right),$$

which $\{J_{ij}\}$ and $\{h_i\}$ are randomly chosen according to a certain probability distribution. We find first order phase transitions in terms of the spin glass order parameter $q := 1/N\sum_i\langle\hat{\sigma}_i^z\rangle^2$ for individual realization of disorders. Those first order transitions are strongly sample-dependent, resembling behaviors of physical quantities within the spin glass phase with non-self averaging properties. However, no singularity is found for the sample-averaged spin glass order parameter $\overline{q}$, nor for other thermodynamic quantities such as the specific heat, spin glass susceptibility etc. This indicates that there is no spin glass phase for this model.
Thus, the fidelity susceptibility (quantum SLD-Fisher information) χ_{F}, which is a quantity inspired from quantum information theory, is measured. We see that the sample-averaged fidelity susceptibility $\overline{\chi_{\mathrm{F}}}$ has a diverging point, which comes before the individual first order transitions. This suggests that a novel quantum phase detected by $\overline{\chi_{\mathrm{F}}}$ induces the first order transitions causing the failure of quantum annealing.

[1] J. Takahashi and K. Hukushima, arXiv:1612.08554 (2016).