

Third law of thermodynamics as a single inequality

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The third law of thermodynamics in the form of the unattainability principle states that exact ground-state cooling requires infinite resources. In this contribution, based on Ref. [1], we will focus on quantifying in full generality the expenditure of fuel –represented by arbitrary systems out of equilibrium– that are needed for approximate cooling. Our scenario is similar to the one considered in algorithmic cooling, but here we treat the full thermodynamics of the problem by allowing for resources with non-trivial Hamiltonians and accounting for the energy conservation of the total process.

The task of cooling that we are considering can be phrased as finding a cooling protocol between an arbitrary resource –playing the role of the fuel– described by the state and Hamiltonian ρ_R and H_R respectively, and a target system described at the end of the process by ρ_S and H_S so that ρ_S is a thermal state with temperature T_S as low as possible. The protocol is done in an environment at inverse temperature β . We are concerned with the ultimate bounds on T_S as a function of the initial resource and β , hence we allow as cooling protocol for any possible energy-preserving unitary on the compound. Our main result is that, in the limit of sufficiently small T_S , a single condition is sufficient and necessary for the cooling protocol to exist. In particular, we find a state function \mathcal{V}_β so that the cooling protocol – under reasonable assumptions – is possible iff

$$\mathcal{V}_\beta(\rho_R, H_R) \geq \mathcal{V}_\beta(\rho_S, H_S). \quad (1)$$

This formulation of the third law, as the requirement that a state function decreases in the process, has a profound explanatory power, since it puts it in a form that is analagous in various ways to the second law. First, note that the second law, in many of its equivalent formulations, can be put, similarly to Eq. (1), as the decrease of a state function – the non-equilibrium free energy – when a system evolves together with an environment at inverse temperature β . Secondly, we find that \mathcal{V} displays a surprising symmetry with the free energy: both functions are defined as the quantum relative entropy S between the same pair of quantum states, in such a way $\mathcal{V} \propto S(\rho||\sigma)$ while the non-equilibrium free energy is proportional to $S(\sigma||\rho)$.

[1] H. Wilming and R. Gallego arXiv:1701.07478 (2017).