Fluctuation-induced forces in confined free and imperfect Bose gases

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Ideal and interacting Bose gases confined to a d-dimensional film of size $\infty^{d-1} \times D$ with periodic (P), antiperiodic (A), Dirichlet-Dirichlet (DD), Neumann-Neumann (NN), and Robin (R) boundary conditions (BCs) are investigated for $2 < d < 4$. The scaling functions $\Upsilon_d^{\rm BC}(x_\lambda = D/\lambda_{\rm th}, x_\xi = D/\xi)$ of the D-dependent residual part $\varphi_{d,{\rm res}}^{\rm BC}(T,\mu,D) = D^{-(d-1)}\Upsilon_d^{\rm BC}(x_\lambda, x_\xi)$ of the grand potential per cross-sectional area and k_BT and their analogs for the fluctuation-induced ("Casimir") force $\beta\mathcal{F}_C^{\rm BC} = -\partial\varphi_{d,{\rm res}}^{\rm BC}/\partial D$ are determined for the ideal gas case with these BCs, where $\lambda_{\rm th}$ and ξ are the thermal de-Broglie wavelength and the bulk correlation length, respectively. The associated limiting scaling functions $\Theta_d^{\rm BC}(x_\xi) \equiv \Upsilon_d^{\rm BC}(\infty, x_\xi)$ describing the critical behavior at the bulk condensation transition are shown to agree with those previously determined from a massive free $O(2)$ theory for BC $=$ P, A, DD, DN, NN. For $d = 3$, they are expressed in closed analytical form in terms of polylogarithms. The analogous scaling function $\Theta_d^{\rm R}(x_\xi, c_1D, c_2D)$ under the RBCs $(\partial_z - c_1)\phi|_{z=0} = (\partial_z + c_2)\phi|_{z=D} = 0$ with $c_1 \geq 0$ and $c_2 \geq 0$ is also determined. The corresponding scaling functions $\Upsilon_{\infty,d}^{\rm P}(x_\lambda, x_\xi)$ and $\Phi_{\infty,d}^{\rm P}(x_\xi)$ for the imperfect Bose gas are shown to agree with those of the interacting Bose gas with n internal degrees of freedom in the limit $n \to \infty$. Hence for $d = 3$, $\Phi_{\infty,d}^{\rm P}(x_\xi)$ is known exactly in closed analytic form. To account for the breakdown of translation invariance in the direction perpendicular to the boundary planes implied by free BCs such as DDBCs, a modified imperfect Bose-gas model is introduced that corresponds to the limit $n \to \infty$ of this interacting Bose gas. Numerically and analytically exact results for the scaling function $\Theta_{\infty,3}^{\mathbb{DD}}(x_\xi)$ therefore follow from those of the $O(2n)$ ϕ^4 model for $n \to \infty$.