Justification of statistical ensembles from thermodynamic transitions

P. Boës, H. Wilming, J. Eisert, R. Gallego

Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany.

Maximum-entropy ensembles have proven to be a very useful primitive from which emergent thermodynamic properties can be derived. Several approaches have been put forward in order to justify, from minimal assumptions, the use of these ensembles as an statistical description of most systems. However, there is to date no generally accepted justification, mainly because most arguments are based on a notion of typicality according to some measure on the state space. Here, we provide a new approach to justify the use of maximum-entropy ensembles. We look at the set of possible transitions that a system can undergo together with an environment, when one only has partial information about both system and environment.

In particular, we ask which final states of a given quantum system we can reach deterministically if the only information that we have about the initial states of *both* the system and the environment are their mean energies. Given a system Hamiltonian H, if a thermodynamic transition from *every* quantum state of initial mean energy e to the *same* final state ρ_f is possible, we write $(e, H) \overset{\text{part. inf.}}{\longrightarrow} \rho_f$. We compare this with the possible thermodynamic transitions that can be implemented one a system if one has full knowledge of both its initial state and that of the environment, $\rho \overset{\text{full inf.}}{\longrightarrow} \rho_f$. Our main result is that, for all e, H

$$(e, H) \overset{\text{part. inf.}}{\longrightarrow} \rho_f \Leftrightarrow \gamma_e(H) \overset{\text{full inf.}}{\longrightarrow} \rho_f, \tag{1}$$

where $\gamma_e(H)$ is the (canonical) maximum-entropy ensemble compatible with the partial information. (1) says that the canonical ensemble is *operationally equivalent* to states of partial information in the sense that a thermodynamic transition can be induced on the latter states if and only if it can be induced on the system in the corresponding canonical ensemble state.

Note that comparing partial information states and ensemble states in terms of their possible thermodynamic transitions is very natural, because most thermodynamic tasks as well as the laws of thermodynamics can be formulated in terms of state transitions, so that our results allow us to re-derive many standard results of phenomenological, such as work extraction bounds or the Clausius inequality, without requiring the usual assumption that the system is in the canonical ensemble state. In this sense, our results justify the overwhelming success of maximum-entropy ensembles to derive thermodynamic laws and provides a derivation that neither relies on any probability measure nor considerations about typical states.